May 27, 1886.

Professor STOKES, D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read :---

I. "Family Likeness in Eye-colour." By FRANCIS GALTON, F.R.S. Received May 10, 1886.

My inquiry into Family Likeness in Stature (*ante*, p. 42) enabled me to define, in respect to that particular quality, the relation in which each man's peculiarity stands to those of each of his ancestors. The object of the present memoir is to verify that relation with respect to another quality, namely, eye-colour.

Speaking of heritage, independently of individual variation, and supposing female characteristics to be transmuted to their male equivalents, I showed (1) that the possession of each unit of *peculiarity* in a man [that is of difference from the average of his race] when the man's ancestry is unknown, implies the existence on an average of just one-third of a unit of that peculiarity in his "mid-parent," and, consequently, in each of his parents; also just one-third of a unit in each of his children; (2) that each unit of peculiarity in each ancestor taken singly, is reduced in transmission according to the following average scale:—from a parent, to $\frac{1}{4}$; from a grandparent, to $\frac{1}{16}$; from a great-grandparent, to $\frac{1}{16}$, and so on.

Stature and eye-colour are not only different as qualities, but they are more contrasted in hereditary behaviour than perhaps any other simple qualities. Speaking broadly, parents of different statures transmit a blended heritage to their children, but parents of different eye-colours transmit an alternative heritage. If one parent is as much taller than the average of his or her sex as the other parent is shorter, the statures of their children will be distributed in much the same way as those of parents who were both of medium height. But if one parent has a light eye-colour and the other a dark eye-colour, the children will be partly light and partly dark, and not medium eye-coloured like the children of medium eye-coloured parents. The blending in stature is due to its being the aggregate of the quasiindependent inheritances of many separate parts, while eye-colour appears to be much less various in its origin. If then it can be shown, as I shall be able to do, that notwithstanding this two-fold difference between the qualities of stature and eye-colour, the shares of hereditary contribution from the various ancestors are in each case alike, we may with some confidence expect that the law by which those hereditary contributions are governed will be widely, and perhaps even universally, applicable.

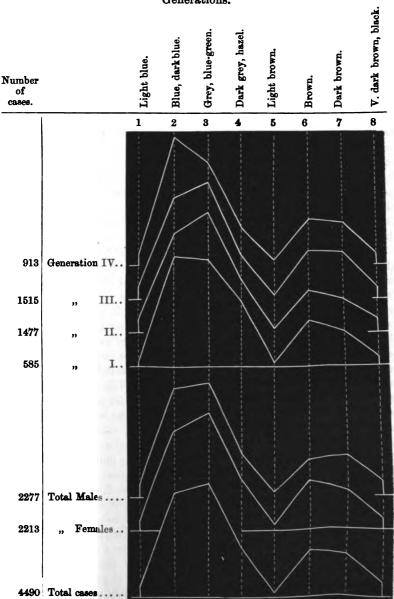
Data.-My data for hereditary eye-colour are drawn from the same collection of "Records of Family Faculties" ("R.F.F.") as those upon which the above-mentioned inquiries into hereditary stature were principally based. I then analysed the general value of these data in respect to stature, and showed that they were fairly trustworthy. I think they are somewhat more accurate in respect to eye-colour, for which family portraits have often furnished direct information, while indirect information has been in other cases obtained from locks of hair that were preserved in the family as mementos. I have also been able to collate some of my results with those lately published by M. Alphonse de Candolle,* who instituted an inquiry that has in many particulars, though not in the main object of the present memoir, covered the same ground as my own, and which was of course founded on an entirely different collection of data. My conclusions in respect to those particulars, of which only a few find place here, are generally corroborated by his.

Persistence of Eye-colour in the Population.-The first subject of our inquiry must be into the existence of any slow change in the statistics of eve-colour in the population that might have to be taken into account before drawing hereditary conclusions. For this purpose I sorted the data, not according to the year of birth, but according to generations, as that method of procedure best accorded with the particular form in which all my R.F.F. data are compiled. Those persons who ranked in the Family Records as the "children" of the pedigree, were counted as generation I; their parents, uncles and aunts, as generation II; their grandparents, great uncles, and great annts, as generation III; their great grandparents, and so forth, as generation IV. No account was taken of the year of birth of the "children," except to learn their age; consequently there is much overlapping of dates in successive generations. We may, however, safely say, that the persons in generation I are quite different from those in generation III, and the persons in II from those in IV. I had intended to exclude all children under the age of eight years, but in this particular branch of the inquiry, I fear that some cases of young children have been accidentally included. I would willingly

* Hérédité de la couleur des yeux dans l'espèce humaine," par M. Alphonse de Candolle. "Arch. Sc. Phys. et Nat. Genève," Aug. 1884, 3rd period, vol. xii, p. 97. Table I.-Frequency of Different Eye-colours in Four Successive Generations.

	8. Very dark brown. Black.	99.9 99.9 100.0 100.0	100.0	99.9 99.9 99.9	6-66	8888	100 -2
	Black.				<u> </u>	13333	I ≍_
	amond froh mov 8	2 8 4 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9	9.8	1 3 5 7 5 7 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	2 .9	∞∞ 4•ø	8 · 8
	7. Дягк ргочп.	9.5 12.6 10.0 10.4	10.8	12:9 12:5 8:2 7:7	10.5	11 12 9	10.7
Percentages.	6. Вючп.	8.4 10·1 11·2 12·4	10.4	15 ·6 13 ·5 10 ·6 11 ·5	12.7	12 12 11	11.6
ercen	5. Light brown.	0.4 0.4 0.3	9.0	001 4094 740	8.0		2.0
P	4. Dark grey. Hazel.	8 ·6 10 ·9 14 ·6 15 ·7	12.3	10-7 12·1 13·3 19·2	13 • 1	11 19	12.7
	3. Grey. Blue-green.	29 4 30 1 31 8 27 4	30.2	25 ·3 32 ·5 32 ·8 28 ·7	30.6	27 31 28 28	30-4
	2. Blue. Dark blue.	38·2 30·3 22·5 28·9	29.3	29.3 23.3 28.6 27.3	26.8	28 28 28	28.1
	l. Light blue.	22241 2400	2.9	2.19	2.5	~~~~	2.7
	Totals.	463 773 742 299	2277	450 742 735 286	2213	913 1515 1477 585	4490
, g	8. Very dark brown. Віяск.	12 24 36 9	81	19 17 24 5	65	14 14 14 14 14 14 14 14 14 14 14 14 14 1	146
observe	7. Дягіг ргомп.	44 97 74 31	246	50 33 88 50 33 88 50 33 88	233	102 190 134 53	479
-colour	6. Brown.	39 79 83 87	238	70 100 78 33	281	109 179 161 70	619
f eye	5. Light brown.	10001	14	01 00 - 1 10	17	40118	81
No. of cases of eye-colour observed.	4. Dark grey. Hazel.	40 84 108 47	279	48 89 89 89 80 89 80 80 80 80 80 80 80 80 80 80 80 80 80	290	88 173 206 102	669
No. of	3. Grey. Blue-green.	136 233 236 82	489	114 241 241 82	849	240 474 477 477 164	1866
	2. Blue. Dark blue.	177 234 167 89	499	132 173 210 78	593	809 407 377 167	1260
	l. Light dlue.	13 19 80 8	66	22 21 6	56	20 81 81 81	181
	Sex and the No. of the generation.	$Males \begin{cases} IV \\ III \\ II \\ II \\ I \end{pmatrix}$	General	$F_{emales} \begin{cases} IV \\ III \\ II \\ II \\ I \\ I \\ I \\ I \\ I $	General	Males IIV and III Females I	General

[May 27,



Percentages of the Various Eye-colours in Four Successive Generations.

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have taken a later limit than eight years, but could not spare the data that would in that case have been lost to me.

A great variety of terms are used by the various compilers of the "Family Records" to express eye-colours. I began by classifying them under the following eight heads:—1, light blue; 2, blue, dark blue; 3, grey, blue-green; 4, dark grey, hazel; 5, light brown; 6, brown; 7, dark brown; 8, black. Then I constructed Table I.

The accompanying diagram will best convey the significance of the figures in Table I. Considering that the headings for different eyecolours are eight in number, the observations are far from being sufficiently numerous to justify us in expecting clean results; nevertheless the curves come out surprisingly well, and in accordance with one another. There can be little doubt that the change, if any, during four successive generations is very small, and much smaller than mere memory is competent to take note of. I therefore disregard a current popular belief in the existence of a gradual darkening of the population, and shall treat the eye-colours of those classes of the English race who have contributed the records, as statistically persistent during the period under discussion.

The concurrence of the four curves for the four several generations affords some internal evidence of the trustworthiness of the data. For supposing we had curves that exactly represented the true eye-colours for the four generations, they would either be concurrent or they would not. If concurrent, the errors in the R.F.F. curves must have been so curiously distributed as to preserve the concurrence. If not, the errors must have been so curiously distributed as to neutralise the non-concurrence. Both of these suppositions are improbable, and we must conclude that the curves really agree, and that the R.F.F. errors are not large enough to spoil the agreement. The much closer concurrence of the two curves, derived respectively from the whole of the male and the whole of the female data, and the still more perfect form of the curve derived from the aggregate of all the cases, are additional evidences in favour of the goodness of the data on the whole.

Fundamental Eye-colours.—It is agreed among most writers on the subject (cf. A. de Candolle) that the one important division of eyecolours is into the light and the dark. The medium tints are not numerous, and they may have four distinct origins. They may be hereditary with no notable variation, they may be varieties of light parentage, they may be varieties of dark parentage, or they may be blends. These medium tints are classed in my list under the heading "4. Dark grey, hazel," and they form only 12.7 per cent. of all the observed cases. It is common in them to find the outer portion of the iris to be of a dark grey colour, and the inner of a hazel. The proportion between the grey and the hazel varies in different cases, and the eye-colour is then described as dark grey or as hazel, accord1886.]

ing to the colour that happens most to arrest the attention of the observer. For brevity, I will henceforth call all intermediate tints by the one name of hazel.

I will now investigate the history of those hazel eyes that are variations from light or from dark respectively, or that are blends between them. It is reasonable to suppose that the residue which were inherited from hazel-eyed parents arose originally either as variations or as blends, and therefore the result of the investigation will enable us to assort the small but troublesome group of hazel eyes in an equitable proportion between light and dark, and thus to simplify our inquiry.

The family records include 168 families of brothers and sisters, counting only those who were above eight years of age, in whom one member at least had hazel eyes. The total number of these brothers and sisters is 948, of whom 302 or about one-third have hazel eyes. For distinction I will describe these as "hazel-eyed families"; not meaning thereby that all the children have that peculiarity, but only some of them. The eye-colours of all the 336 parents are given in the records, but only those of 449 of the grandparents, whose number would be 672, were it not for a few cases of cousin marriages. Thus I have information concerning about only two-thirds of the grandparents, but this will suffice for our purpose. The results are given in Table II.

	Total		Observed	•	Percentages.			
	C8868.	Light.	Hazel.	Dark.	Light.	Hazel.	Dark.	
General population III, Grandparents II, Parents I, Children	4490 449 336 948	2746 267 165 430	569 61 85 302	1175 121 86 216	61 ·2 60 49 45	12 ·7 13 25 32	26 ·1 27 26 23	

Table II.—The Descent of Hazel-eyed Families.

It will be observed that the distribution of eye-colour among the grandparents of the hazel-eyed families is nearly identical with that among the population at large. But among the parents there is a notable difference; they have a decidedly smaller percentage of light eye-colour and a slightly smaller proportion of dark, while the hazel element is nearly doubled. A similar change is superadded in the next generation. The total result in passing from generation III to I, is that the percentage of the light eyes is diminished from 60 or 61 to 45, therefore by one quarter of its original amount, and that

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the percentage of the dark eyes is diminished from 26 or 27 to 23, that is to about one-eighth of its original amount, the hazel element in either case absorbing the difference. It follows that the chance of a light-eyed parent having hazel offspring, is about twice as great as that of a dark-eyed parent. Consequently, since hazel is twice as likely to be met with in any given light-eyed family as in a given dark-eyed one, we may look upon two-thirds of the hazel eyes as being fundamentally light, and one-third of them as fundamentally dark. I shall allot them rateably in that proportion between light and dark, as nearly as may be without using fractions, and so get rid of them. M. Alphonse de Candolle has also shown from his data, that yeux gris (which I take to be the equivalent of my hazel) are referable to a light ancestry rather than to a dark one, but his data are numerically insufficient to warrant a precise estimate of the relative frequency of their derivation from each of these two sources.

Heredity of Light and Dark Eye-colour.-In the following discussion I shall deal only with those family groups of children in which the eyecolours are known of the two parents and of the four grandparents. There are altogether 211 of such groups, containing an aggregate of 1023 children. They do not, however, belong to 211 different family stocks, because each stock which is complete up to the great grandparents inclusive (and I have fourteen of these) is capable of yielding three such groups. Thus, group 1 contains a, the "children;" b, the parents; c, the grandparents. Group 2 contains a, the father of the "children," his brothers and his sisters; b, the parents of the father; c, the grandparents of the father. Group 3 contains the corresponding selections on the mother's side. Other family stocks furnish two groups. Out of these and other data, Tables III and IV have been made. In Table III I have classified the families together whose two parents and four grandparents present the same combination of eye-colour, no class, however, being accepted that contains less than twenty children. These data will enable us to test the average correctness of the law I desire to verify, because many persons and many families appear in the same class, and individual peculiarities tend to disappear. In Table IV I have separately classified on the same system all the families, 78 in number, that consist of six or more children. These data will enable us to test the trustworthiness of the law as applied to individual families. It will be seen from my way of discussing them, that smaller families than these could not be advantageously dealt with.

Table III.—Sixteen Groups of Families, those being grouped together in whom the distribution of Light, Hazel, and Dark Eye-colour among their Parents and Grandparents is alike. Each Group contains at least Twenty Brothers or Sisters.

	Eye-colours of the						Number of the light eye- coloured children.			
Parents. Grandparents.			Total children.	Observed	Calculated.					
Light.	Hazel.	Dark.	Light.	Hazel.	Dark.		Observed.	I.	11.	111.
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	··· ·· ·· ·· ·· ·· ·· ·· ·· ·· ·· ··	··· ·· ·· ·· ·· ·· ·· ·· ·· ·· ·· ·· ··	4 3 3 2 2 3 3 2 2 2 2 2 2 1 1 1 1	 1 1 2 1 1	$\begin{array}{c} \ddots \\ 1 \\ 2 \\ \\ 1 \\ \\ 2 \\ 1 \\ 1 \\ 3 \\ 2 \\ 2 \\ 2 \\ 2 \\ \end{array}$	183 53 92 27 22 62 42 31 49 31 76 66 27 20 22 24	174 46 88 26 11 52 80 28 85 25 45 80 15 9 8 9 9	161 47 81 24 6 48 33 24 38 24 38 24 44 38 16 12 13 14 623	163 44 67 18 12 51 31 24 28 21 55 38 21 55 38 18 8 11 12 601	172 48 79 22 6 51 32 20 34 23 46 35 16 9 11 10 614

Mr. F. Galton.

Eye-colours of the							Num c	iber of a	the ligh childre	t eye- n.
	Parents	its. Grandparents.		Grandparents.		Total child- ren.	ОЪ-	C	alculate	×d.
Light.	Hazel.	Dark.	Light.	Hazel.	Dark.		served.	I.	11.	III.
222222222222222222222222222222222222222	$ \begin{array}{c} \cdots \\ $	······································	4444444444444444444383333333222222321111333333332	$ \begin{array}{c} \cdots \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ 2 \\ 1 $	$\cdots \cdots $	666677777888888812710127899991071060178699611676789119117	6666577777888877727426899707960452113.466670676	5555666667777777106806209999828382475717075206065 566666777777106806209999828382475717075206065	33832222221111111783018666347077641550607651604 55556666677777715800186666347077641550607651604	55666666655555341909777628036825225897642954 6677777771691066777762803682525225897642954

Table IV.—78 Separate Families, each of not less than 6 Brothers or Sisters.

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	Table	IV	-continued.
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	Eye-colours of the						Num	t ey e- n.		
	Parents. Grandparents.				Grandparents.			(Children	L.
Light.	Hazel.	Dark.	Light.	Hazel.	Dark.		served.		п.	111.
$ \begin{array}{c} 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ $		$ \begin{array}{c} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ 1111111111$	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	$ \begin{array}{c} 2 \\ 2 \\ \\ 1 \\ 1 \\ 3 \\ 1 \\ \\ \\ \\ \\ 1 \\ 2 \\ 1 \\ \\ 4 \\ \end{array} $	····222211 ···22211 ···22211 ···2222112333213 ···	$9 \\ 11 \\ 6 \\ 8 \\ 9 \\ 6 \\ 10 \\ 9 \\ 8 \\ 7 \\ 6 \\ 7 \\ 8 \\ 8 \\ 9 \\ 9 \\ 6 \\ 6 \\ 8 \\ 10 \\ 14 \\ 7 \\ 7 \\ 8 \\ 8 \\ 6 \\ 6 \\ 9 \\ 13 \\ 7 \\ 7 \\ 8 \\ 8 \\ 6 \\ 6 \\ 9 \\ 13 \\ 7 \\ 7 \\ 8 \\ 8 \\ 6 \\ 6 \\ 9 \\ 13 \\ 7 \\ 7 \\ 7 \\ 8 \\ 8 \\ 8 \\ 9 \\ 9 \\ 6 \\ 6 \\ 9 \\ 13 \\ 7 \\ 7 \\ 7 \\ 8 \\ 8 \\ 8 \\ 9 \\ 9 \\ 6 \\ 6 \\ 9 \\ 13 \\ 7 \\ 7 \\ 7 \\ 8 \\ 8 \\ 9 \\ 9 \\ 6 \\ 6 \\ 9 \\ 13 \\ 7 \\ 7 \\ 7 \\ 8 \\ 8 \\ 9 \\ 9 \\ 6 \\ 6 \\ 9 \\ 13 \\ 7 \\ 7 \\ 7 \\ 8 \\ 8 \\ 9 \\ 9 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\$	9164576945343654655342953443333482	$\begin{array}{c} 7 \cdot 6 \cdot 7 \\ 4 \cdot 7 \cdot 2 \cdot 0 \cdot 7 \\ 4 \cdot 5 \cdot 1 \cdot 6 \cdot 6 \cdot 6 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 6 \cdot 8 \cdot 1 \cdot 1 \\ 4 \cdot 4 \cdot 4 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 6 \cdot 8 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 6 \cdot 6 \cdot 5 \cdot 8 \cdot 0 \cdot 1 \cdot 5 \\ 4 \cdot 4 \cdot 4 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 6 \cdot 8 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 6 \cdot 6 \cdot 5 \cdot 8 \cdot 0 \cdot 1 \cdot 5 \\ 10 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 6 \cdot 8 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 6 \cdot 6 \cdot 5 \cdot 8 \cdot 0 \cdot 1 \cdot 5 \\ 10 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 6 \cdot 8 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 5 \cdot 6 \cdot 5 \cdot 8 \cdot 0 \cdot 1 \cdot 5 \\ 10 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 6 \cdot 8 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 5 \cdot 6 \cdot 5 \cdot 8 \cdot 0 \cdot 1 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 6 \cdot 8 \cdot 1 \cdot 1 \cdot 1 \cdot 5 \cdot 6 \cdot 5 \cdot 8 \cdot 0 \cdot 1 \cdot 5 \cdot 5$	69546107512418886664467073633140886 54654655556634584433321408654	$5 \cdot 791$ $4 \cdot 15244$ $4 \cdot 56 \cdot 4765$ $4 \cdot 999$ $4 \cdot 15244$ $4 \cdot 999$ $5 \cdot 524$ $4 \cdot 999$ $5 \cdot 5222$ $5 \cdot 5222$ $5 \cdot 5222$ $5 \cdot 566666$ $6 \cdot 6647$ $4 \cdot 74$ $5 \cdot 566$ $5 \cdot 6666$ $5 \cdot 437$ $4 \cdot 999$ $5 \cdot 5222$ $5 \cdot 5222$ $5 \cdot 566$ $5 \cdot 6666$ $6 \cdot 47$ $4 \cdot 52$ $5 \cdot 5222$ $5 \cdot 5222$ $5 \cdot 56$ $5 \cdot 6666$ $6 \cdot 66$ $4 \cdot 74$ $5 \cdot 522$ $5 \cdot 5222$ $5 \cdot 5222$ $5 \cdot 56$ $5 \cdot 6666$ $6 \cdot 66$ $4 \cdot 74$ $5 \cdot 5222$ $5 \cdot 5222$ $5 \cdot 56$ $5 \cdot 666$ $5 \cdot 66$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 56$ $5 \cdot 666$ $5 \cdot 66$ $5 \cdot 437$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 56$ $5 \cdot 666$ $5 \cdot 66$ $5 \cdot 437$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 56$ $5 \cdot 66$ $5 \cdot 66$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 56$ $5 \cdot 66$ $5 \cdot 66$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 522$ $5 \cdot 52$ $5 \cdot 52$ $5 \cdot 52$ $5 \cdot 56$ $5 \cdot 66$ $5 \cdot 66$ $5 \cdot 522$ $5 \cdot 52$ $5 \cdot 52$ 5

It will be noticed that I have not printed the number of dark-eyed children in either of these tables. They are implicitly given, and instantly to be found by subtracting the number of light-eyed children from the total number of children. Nothing would have been gained by their insertion, while compactness would have been sacrificed.

The entries in the tables are classified, as I said, according to the various combinations of light, hazel, and dark eye-colours in the parents and grandparents. There are 6 different possible combinations among the two parents, and 15 among the four grandparents; making 90 possible classes altogether. The number of observations are of course by no means evenly distributed among the classes. I have no returns at all under more than half of them, while the entries of two light-eyed parents and four light-eyed grandparents are proportionately very numerous. (I shall not here discuss the question of marriage selection in respect to eye-colour, which is a less simple statistical question than it may appear to be at first sight.)

Calculation.—I have now to show how the expectation of eyecolour among the children of a given family is to be calculated on the basis of the law laid down for stature, so that those calculations of the probable distribution of eye-colours may be made, which fill the three last columns of Tables III and IV, which are headed I, II, and III, and which are placed in juxtaposition with the observed facts as entered in the column headed "Observed." These three columns contain calculations based on data limited in three different ways, in order the more thoroughly to test the applicability of the law that it is desired to verify. Column I contains calculations based on a knowledge of the parents only; III contains those based on a knowledge both of the parents and of the grandparents, and of them only.

I. Eye-colours given of the two parents-

Let the letter M be used as a symbol to signify the person for whom the expected heritage is to be calculated. Let P stand for the words "a parent of M;" G_1 for "a grandparent of M;" G_2 for "a great-grandparent of M," and so on.

Now suppose that the amount of the peculiarity of stature possessed by P is equal to r, and that nothing whatever is known with certainty of any of the ancestors of M except P. We have seen* that though nothing may be actually known, yet that something definite is implied about the ancestors of P, namely, that each of his two parents (who stand in the order of relationship of G_1 to M) will on the average possess $\frac{1}{3}r$. Similarly that each of the four grandparents of P(who stand in the order of G_2 to M) will on the average possess $\frac{1}{3}r$, and so on. Again we have seen that P, on the average, transmits to M only $\frac{1}{4}$ of his peculiarity; that G_1 transmits only $\frac{1}{16}$; G_2 only $\frac{1}{64}$, and so on. Hence the aggregate of the heritages that may be expected to converge through P upon M, is contained in the following series:—

$$r\left\{\frac{1}{4} + 2\left(\frac{1}{3} \times \frac{1}{2^{4}}\right) + 4\left(\frac{1}{9} + \frac{1}{2^{6}}\right) + & \text{åc.}\right\}$$
$$= r\left\{\frac{1}{2^{3}} + \frac{1}{2^{3} \cdot 3} + \frac{1}{2^{4} \cdot 3^{2}} + & \text{åc.}\right\} = r \times 0.30.$$
$$* \quad Ante, p. 42 (No. 242).$$

That is to say, each parent must in this case be considered as contributing 0.30 to the heritage of the child, or the two parents together as contributing 0.60, leaving an indeterminate residue of 0.40 due to the influence of ancestry about whom nothing is either known or implied, except that they may be taken as members of the same race as M.

In applying this problem to eye-colour, we must bear in mind that a given fractional chance that each member of a family will inherit either a light or a dark eye-colour, must be taken to mean that that same fraction of the total number of children in the family will probably possess it. Also, as a consequence of this view of the meaning of a fractional chance, it follows that the residue of 0.40 must be rateably assigned between light and dark eye-colour, in the proportion in which those eye-colours are found in the race generally, and this was seen to be (see Table II) as $61\cdot2:26\cdot1$; so I allot 0.28 out of the above residue of 0.40 to the heritage of light, and 0.12 to the heritage of dark. When the parent is hazel-eyed I allot $\frac{2}{3}$ of his total contribution of 0.30, *i.e.*, 0.20 to light, and $\frac{1}{3}$, *i.e.*, 0.10 to dark. These chances are entered in the first pair of columns headed I, in Table V.

	Data limited to the eye-colours of the										
Contribution to the heritage from each.	2 рал	rents.	4 grand	parents.	2 parents and 4 grandparents. III.						
]	[.	I	[.							
	Light.	Dark.	Light.	Dark.	Light.	Dark.					
Light-eyed parent Hazel-eyed parent Dark-eyed parent	0 · 30 0 · 20 	 0 · 10 0 • 3 0	 	 	0·25 0·16 	0 ∙09 0 ∙25					
Light-eyed grandparent. Hazel-eyed grandparent Dark-eyed grandparent.	••• ••	•• •• ••	0·16 0·10 	0 ^{.06} 0.16	0 ·08 0 ·05 	 0 ∙03 0 ∙08					
Residue, rateably as- signed	0 • 28	0 · 12	0 • 25	0.11	0 .12	0 .06					

Table V.

		I.			II.		III.			
Ancestry and their eye-colours.	No. about whom data exist.	Contribute to		. about whom ata exist.	Contribute to		sbout whom sta exist.		ribute o	
	No. a dai	Light.	Dark.	No. s da	Light.	Dark.	No. s dat	Light.	Dark.	
Light-eyed parents Hazel-eyed parents Dark-eyed parents	2 	0 ·60 	••	 	••	••	 1 1	0.16 	 0·09 0·25	
Light - eyed grand- parents		••	••	1	0 · 16 0 ·20	 0·12	1	0 ·08 0 ·10	 0.06	
Dark - eyed grand- parents	••	••		1		0 12 0 · 16	1		0·08	
Residue, rateably as- signed		0 ·28	0 · 12		0 · 25	0.11		0.15	0 -06	
Total contributions	••	0.88	0 · 12		0 •61	0 · 39		0.46	0.54	
•		1.00			1.	00		1.	00	

Table VI.—Example of one Calculation in each of the 3 Cases.

The pair of columns headed I in Table VI shows the way of summing the chances that are given in the columns with a similar heading in Table V. On the method there shown I calculated all the entries that appear in the columns with the heading I in Tables III and IV.

II. Eye-colours given of the four grandparents-

Suppose r to be possessed by G_1 and that nothing whatever is known with certainty of any other ancestor of M. Then it has been shown that the child of G_1 (that is P) will possess $\frac{1}{3}r$; that each of the two parents of G_1 (who stand in the relation of G_2 to M) will also possess $\frac{1}{3}r$; that each of the four grandparents of G_1 (who stand in the relation of G_8 to M) will possess $\frac{1}{9}r$, and so on. Also it has been shown that the shares of their several peculiarities that will on the average be transmitted by P, G_1 , G_2 , &c., are $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, &c., respectively. Hence the aggregate of the probable heritages from G_1 are expressed by the following series :—

$$r\left\{\frac{1}{3} \times \frac{1}{2^{2}} + 1 \times \frac{1}{2^{4}} + \frac{1}{3} \times 2 \times \frac{1}{2^{6}} + \frac{1}{9} \times 4 \times \frac{1}{2^{8}} + \text{ &c.}\right\}$$
$$= r\left\{\frac{1}{12} + \left(\frac{1}{2^{4}} + \frac{1}{3 \times 2^{6}} + \frac{1}{3^{9} \times 2^{6}} + \text{ &c.}\right)\right\} = \frac{1}{12} + \frac{3}{40} = 0.16.$$

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So that each grandparent contributes on the average 0.16 (more exactly 0.1583) to the heritage of M, and the four grandparents contribute between them 0.64, leaving 36 indeterminate, which when rateably assigned gives 0.25 to light and 0.11 to dark. A hazel-eyed grandparent contributes, according to the ratio described in the last paragraph, 0.10 to light and 0.06 to dark. All this is clearly expressed and employed in the columns II of Tables V and VI.

III. Eye-colours given of the two parents and four grandparents-

Suppose P to possess r, then P taken alone, and not in connexion with what his possession of r might imply concerning the contributions of the previous ancestry, will contribute an average of 0.25 to the heritage of M. Suppose G₁ also to possess r, then his contribution together with what his possession of r may imply concerning the previous ancestry, was calculated in the last paragraph as $\frac{1}{40} = 0.075$. For the convenience of using round numbers I take this as 0.08. So the two parents contribute between them 0.50, the four grandparents together with what they imply of the previous ancestry contribute 0.32, being an aggregate of 0.82, leaving a residue of 0.18 to be rateably assigned as 0.12 to light, and 0.6 to dark. A hazeleyed parent is here reckoned as contributing 0.16 to light and 0.9 to dark; a hazel-eyed grandparent as contributing 0.5 to light and 0.3 to dark. All this is tabulated in Table V, and its working explained by an example in the columns headed III of Table VI.

Results.—A mere glance at Tables III and IV will show how surprisingly accurate the predictions are, and therefore how true the basis of the calculations must be. Their *average* correctness is shown best by the totals in Table III, which give an aggregate of calculated numbers of light-eyed children under Groups I, II, and III as 623, 601, and 614 respectively, when the observed numbers were 629; that is to say, they are correct in the ratios of 99, 96, and 98 to 100.

Table VII.

Number of Errors of various Amounts in the 3 Calculations of the Numbers of Light Eye-coloured Children in the 78 Families.

		Amou	int of I	Errors.		
Data employed referring to	0.0 to 0.5.	0.6 to 1.1.	1 ·2 to 1 ·7.	1 · 8 to 2 · 3.	2·4 and above.	Total Cases.
I. The 2 parents only II. The 4 grandparents only	19 16	30 28	18 10	5 10	6 14	78 78
III. The two parents and 4 grand- parents	41	17	8	4	8	78

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Their trustworthiness when applied to individual families is shown as strongly in Table IV, whose results are conveniently summarised in Table VI. I have there classified the amounts of error in the several calculations: thus if the estimate in any one family was 3 light-eyed children and the observed number was 4. I should count the error as 1.0. I have worked to one place of decimals in this table, in order to bring out the different shades of trustworthiness in the three sets of calculations, which thus become very apparent. It will be seen that the calculations in Class III are by far the most precise. In more than one-half of those calculations the error does not exceed 0.5, whereas in I and II more than three-quarters of them are wrong to at least that amount. Only one-quarter of Class III are more than 1.1 in error, but somewhere about the half of Classes I and II are wrong to that amount. In comparing I with II, we find I to be slightly, but I think distinctly, the superior estimate. The relative accuracy of III as compared with I and II, is what we should have expected, supposing the basis of the calculations to be true, because the additional knowledge utilised in III, over what is turned to account in I and II, must be an advantage.

Conclusion.—The general trustworthiness of these calculations of the probable proportion of light-eyed and dark-eyed children in individual families, whose ancestral eye-colour is more or less known, is comparable with the chance of drawing a white or a black ball out of a bag in which the relative numbers of white and black balls are the same as those given by the calculation. The larger the proportion of data derived from a certain knowledge of ancestral eye-colours, and not from inferences about them, the more true does the comparison become. My returns are insufficiently numerous and too subject to uncertainty of observation to make it worth while to submit them to a more rigorous analysis, but the broad conclusion to which the present results irresistibly lead, is that the same peculiar hereditary relation that was shown to subsist between a man and each of his ancestors in respect to the quality of stature, also subsists in respect to that of eye-colour.

II. "A General Theorem in Electrostatic Induction, with Application of it to the Origin of Electrification by Friction." By JOHN BUCHANAN, B.Sc., Demonstrator of Physics, University College, London. Communicated by Professor G. CAREY FOSTER, B.A., F.R.S. Received May 13, 1886.

PART I.

This paper contains the results of an investigation into the question: If a dielectric be brought into a field of electric force, and there its