roots of equations. Moreover, there is the paper on the contacts of lines and surfaces of the second order, where the invariant factors of a matrix are recognised, and the system of two quaternary quadratics is considered in detail with reference to the simplest simultaneous reduction of the forms.

Appreciations of Sylvester's character and of the value of his mathematical work have been written by able hands, and it is unnecessary to enlarge upon them here. His egotism was obvious and often amusing, but never offensive; his enthusiasm was refreshing, and though his temper was touchy, he was very generous and kind. As a master of formal analysis he has few equals; the birth of the calculus of invariants occurred just at the right time to attract his attention, and his contributions to this subject alone are enough to make him famous. He had the instincts of an architect, and it is well, on the whole, that he did not always trouble to clear away the chips. The casual remarks scattered about his papers and the fragmentary nature of some of them, help to make the reading of them very stimulating; he takes us into his confidence, shows us how his ideas arose, and gives us hints of unexplored regions. He was eminently original, and spent little time in studying the works of his contemporaries; thus he did not even realise that his theory of reciprocants had been more than anticipated by others, especially by Lie. But any misunderstanding arising from this source must have been long since dissipated, and his place among the great mathematicians of his time is quite secure.

Sylvester's occasional notes on the theory of numbers and his lectures on partitions suggest problems to those who are interested in arithmetic. The present volume, for instance, contains three notes on cubic Diophantine equations, a subject not yet exhausted, though Sylvester's own theory of resideration throws much light upon it. The late Henry Smith once referred to this problem as being one which might be hopefully attacked with the engines of modern analysis; perhaps the appearance of this edition of Sylvester's works may lead to the discovery of a complete theory.

A good example of Sylvester's power of illuminating and drawing general conclusions from the simplest mathematical problem is the note (p. 392) on an elementary geometrical theorem for which no direct proof had been discovered. He observes that the proof may be made to depend on showing that a certain analytical equation has no real root, and suggests that in all such cases where the analytical proof consists in demonstrating the *non-existence* of roots, the geometrical proof must necessarily be indirect, while in other cases the *reductio ad absurdum* may be convenient, but is not necessary. This observation reminds us at once of Gauss's discussion of the division of the circle, and if Sylvester's conjecture is true it gives another case of the curious points of contact that exist between analysis and geometry.

It is not to be expected, or even desired, that many should share Sylvester's keen delight in the beauty of formal analysis; but it is a mistake to discourage those who are inclined to enjoy it, however unpractical parts of the subject may be. Quite apart from other reasons, the study of pure mathematics may be defended, like that of music or chess or painting, from the merely æsthetical side, and this Sylvester does in terms both vigorous and quaint. For example:—

"The fortunate proclaimer of a new outlying planet has been justly rewarded by the offer of a baronetcy and a national pension, which the writer of this wishes him long life and health to enjoy. In the meanwhile, what has been done in honour of the discoverer of a new and inexhaustible region of exquisite analysis?" the latter reference being to Cayley's discovery of the calculus of invariants. Fortunately Cayley was saved in another way from the cares of money-making, and he lived long enough to realise to the full his great reputation among those who would appreciate his work. Sylvester in his early life suffered unjustly from the current prejudice against his race; so far as it was possible this was afterwards atoned for, and it is to be hoped that no bitter feeling was left behind.

G. B. M.

MENTAL AND SOCIAL MEASUREMENTS.

An Introduction to the Theory of Mental and Social Measurements. By Edward L. Thorndike, Professor of Psychology in Teachers' College, Columbia University. Pp. xii+212. (New York: The Science Press, 1904.) Price 1.50 dollars net.

AMERICAN colleges seem more awake than our own to the fact that the newer methods of statistics have made it possible to deal with facts with which they are directly concerned, and to discuss them with far more completeness than was practicable a few years ago. They are making in consequence large collections of anthropometric data to serve as tests of health and development, and for comparisons between colleges. Again, there are more teachers in America than in this country who, appreciating the fact that the above methods have far wider applicability, extend the range of their measurements to psychophysical subjects. They are also eager to deal with purely psychical matters that elude direct measurement but admit of being arranged by mutual comparison into their proper class places, or to utilise a third and still more general method, which deals with such objects as can be sorted into a few distinct classes without regard to their internal arrangement. The author is fully justified in saying that

"The obscurest and most complex traits, such as morality, enthusiasm, eminence, efficiency, courage, legal ability, inventiveness, can be made material for ordinary statistical procedure, the one condition being that the general form of distribution of the trait in question shall be approximately known."

In these circumstances a system of elaborate measurements has come into vogue in many American colleges. Whether the authorities have always planned their measurements wisely, and whether they discuss them adequately and accurately, will not be considered here. The volume is written to direct and to warn, in doing which it reveals some grave blunderings. Unfortunately, it is composed chiefly for those persons who are ignorant of even simple mathematics. The

author is fully conscious of the serious embarrassments of the position he has chosen, but bravely attempts the well-nigh impossible task of overcoming them. Thus he says:—

"If this book were written by a mathematician for the mathematically minded, it would not need to be one fifth as long. If read by such a one it may well seem intolerably clumsy and inelegant."

Whether he succeeds under these difficulties in giving easily intelligible explanations may well be doubted; indeed, his language, though frequently lucid, is often quite the reverse. Still, if the volume were used as a text-book in the hands of an enthusiastic and capable teacher good results might follow, but it requires an optimistic disposition to believe that it would prove more than superficially instructive, if it were intelligible at all, to the mass of ordinary and unassisted readers. The author might, however, claim a higher rank for it than he has done on the ground that it teems with instructive illustrations by which everyone may profit, and that it presents familiar ideas from slightly new points of view, much to the advantage of even well instructed readers.

There is no science more handicapped by cumbrous and repellent terminology than that of the higher statistics. Its ideas are not always intrinsically difficult to grasp, but the phrases by which they are expressed are both ugly and unexpressive. The writer believes that a student, however mathematically minded he may be, would save himself time and annoyance if he prefaced his earliest studies by a few hours of what might be called *kindergarten* exercises with beans, acorns, or the like. By the process of sorting them into arrays and picking out the medians, quartiles, &c., then by measuring them individually and extracting from the measures the remaining statistical constants, he would soon obtain a serviceable familiarity with the more elementary technical terms and the ideas they represent. It would be easy to devise a suitable course that would prove a welcome help to students who are enthusiastic about measurements, and it is to be hoped that the next writer on popular statistics will elaborate one.

The author gives a large number of frequency polygons, derived from a wide variety of data, which are of interest. It is to be wished that attempts were more frequently made to reduce the variously shaped polygons obtained by experience into a few classified types, to append to each type the names of the objects that had been found to conform to it, and to analyse the causes of its shape in each instance. It is difficult to doubt that by so doing some desirable help would be given to the interpretation of any new polygon. It is perfectly true that almost any curve or polygon *may* be built up in various ways by different types of curve or polygons appropriately superposed, but experience alone will tell whether there is not a much greater probability of such and such a type being due to such and such combinations rather than to others. Through these means many hypothetical sources of origin might be found so rare as to be hardly worth considering, and so the field of probable interpretations would be narrowed. Speaking generally, the interpretation of results is a branch of statistics that has hitherto received less attention than it deserves. It is no doubt a great thing to be able to describe groups and to determine correlations between them with precision, but this is not all that is wanted. It is another and even more important achievement to dissect and analyse results and to discover the dominant causes that produced them, but the art of doing this seems as yet inadequately developed and to offer a promising field for research. F. G.

OUR BOOK SHELF.

Practical Chemistry, a Second Year Course. By G. H. Martin, M.A. (N.D.). Pp. 41. (Bradford: G. H. Martin, The Grammar School.) Price 1s.

MR. MARTIN has arranged in an unpretentious form a most excellent syllabus of experiments and examples suitable for boys beginning the study of chemistry.

It is satisfactory to find that, in a school of such high standing as the Bradford Grammar School, the science master has seen the wisdom of devoting a whole year (it is to be hoped it will be extended to a second year) to teaching the simple facts which underlie important principles without recourse to tests and tables.

One suggestion may be offered. If the book is to have a wide circulation, which it certainly deserves, it will be necessary to fill in the outline of experiments, and perhaps to illustrate the results by actual examples, possibly in a companion volume.

Boys cannot be expected to work out details of apparatus in the short time allotted to science during school hours if substantial progress is to be made. No doubt the author has his apparatus set up and gives an appropriate demonstration to the class, but this will not help those teachers who wish to profit by the book unless their technical difficulties are solved for them.

J. B. C.

Retouching. By Arthur Whiting. Pp. xvi+91. (London: Dawbarn and Ward, Ltd., 1904.) Price 1s. net.

IT very often happens that photographic negatives require a certain amount of careful manipulation owing to defects caused by photographic methods, scratches, &c. It is also desired sometimes to eliminate small defects due to slight movement of the object, or to alter or improve portions of the picture to attain a desired end. The author has endeavoured in these few pages to place before the reader the different methods and devices that are in use to cope successfully with the various defects that may be encountered. In the first instance the tools required are described, and the special objects of each explained. The reader is then shown how, in the case of portraits, to preserve the likeness but yet to eliminate the blemishes caused by optical or chemical or other action; he is here introduced in a few words to the elements of facial anatomy. The author has considered it necessary to insert a special chapter on retouching portraits of professionals, in which the main principle to be kept in view is to produce a beautiful face. To attain such an ideal, mouths are reduced, jaws cut down, ears knifed, eyes enlarged, and various other surgical operations performed. Working up draperies, retouching landscapes, preparing prints for the press, and how to make a portable retouching desk, form other topics for treatment. The book should serve as an admirable guide to amateurs, and will be found useful to those who go more especially into this class of work. Numerous illustrations accompany the text.