III. The observed deciles, as given by Mr. Galton, do not seem to me really comparable with the skew-curves. The latter give the form which the observation polygon would probably assume were the number of observations to be indefinitely increased; they give, if desired, a system of theoretical deciles—the deciles that would probably be obtained from an indefinitely large number of observations. They give the mode and its frequency, the limit to the range, the interpolated frequency, &c. The observed deciles give none of these things; they only compare, as a method of representation, with the observed frequency-polygon.

The method of percentiles may, in fact, often be a convenient method of representing observations, but it must be remembered that it is only a method of representation, it offers no theoretical law of distribution or legitimate means of smoothing observations. Its wide adoption to give the results of statistical researches is even to be deprecated, for it frequently causes the omission of the very data upon which a real statistical theory could be developed.

IV.—"*Contributions to the Mathematical Theory of Evolution. Note on Reproductive Selection.*" By Professor KARL PEARSON, M.A., F.R.S., University College, London. Communicated by F. GALTON, F.R.S.

(1) In a recent memoir ("Contributions to the Mathematical Theory of Evolution, III. Regression, Heredity, and Panmixia," now in type for the *Philosophical Transactions*) I have found it necessary to note the difference in mean and variation of a population when (*a*) the individuals of a sex are taken into account once as mates (*b*) when the individuals of a sex are treated as parents or weighted with their fertility. The mean and variation of the population are supposed to be taken with regard to any organ whatever. If such a difference is found to exist between the variation curves for mates and for parents, then there is a correlation between fertility and the organ (or characteristic) measured. Under the action of heredity there will accordingly be a progressive evolution in this organ, unless this evolution be checked by some other factor of progressive change, *e.g.*, natural selection. In my memoir I term this factor of progressive evolution *Reproductive Selection.*[4] Without wishing at present to publish my

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[4] The influence of variation in fertility has been considered by Mr. Romanes under the title of "Physiological Selection." but the idea he expresses by this term appears to me very different from that of reproductive selection. In mathematical language, Mr. Romanes supposes the fertility curve and the correlation surfaces, owing to some cause or other, to become double-humped; they may accordingly be resolved into two components, each corresponding to a distinct species. Physiological selection thus aims at an explanation of the origin of species. Reproductive selection supposes the fertility curve and correlation surfaces to embrace only homogeneous material, and it can accordingly never give

complete work on this subject, I should like to put on record the following conclusions already reached :—

(2) Let any organ in individuals of one sex be selected, and let y be the fertility of an individual, whose organ differs x from the mean organ of all mated individuals. Let M_m be the mean organ for all mates[5], M_p be the mean organ for all parents, *i.e.*, a mate reckoned once for each offspring. Let M_0 be the mean of the offspring for the same or any other organ, taking one or any other number equally from each mated individual, let M_1 be the mean of all offspring. Let σ_m, σ_p, σ_0, σ_1 be the corresponding standard deviations, reckoned from the formula: $\sigma^2 =$ (sum of squares of deviations) $\div$ (number of individuals), and without regard to any special law of variation, such as Laplace's law of errors.

Let r_0 be the coefficient of correlation between parent and offspring, each parent being given only one or, at any rate, an equal number of offspring, *i.e.*, r_0 is the coefficient of pure heredity for the organs in question, supposing fertility to be uniform, or at any rate to have no correlation with the organ or characteristic under investigation. Let ρ be the correlation between fertility and the given organ in the parent, and let v equal the coefficient of variation of fertility in the parent, *i.e.*, if y_m be the mean fertility : $v = \sigma_f / y_m$, where σ_f is the standard deviation of parental fertilities. Let $y' = y - y_m$ be the deviation from mean fertility of the parent with organ x. The values of r_0 and ρ are to be calculated from the formulæ—

$$r_0 = \frac{\text{Sum of (deviation of offspring} \times \text{deviation of parent)}}{\text{Number of pairs of offspring and parent} \times \sigma_0 \times \sigma_m},$$

$$\rho = \frac{\text{Sum of (deviation of mate} \times \text{deviation of mate's fertility)}}{\text{Number of mated pairs} \times \sigma_m \times \sigma_f},$$

where, in r_0, each parent is to be taken only once, or at any rate the same number of times.

Thus, r_0 and ρ are absolutely independent of any special distribution of variation.

Then the following results hold if n be the number of mated pairs :—

$$M_p = M_m + \rho v \sigma_m \quad\ldots\ldots\ldots\ldots \quad \text{(i).}$$

$$\sigma^2_p = \sigma^2_m (1 - \rho^2 v^2) + \frac{S(x^2 y')}{n y_m} \quad\ldots\ldots\ldots\ldots \quad \text{(ii).}$$

$$M_1 = M_0 + r_0 \rho v \sigma_0 \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(iii).}$$

$$\sigma_1^2 = \sigma_0^2 \left(1 - r_0^2 + r_0^2 \frac{\sigma_p^2}{\sigma_m^2}\right) \quad\ldots\ldots\ldots\ldots \quad \text{(iv).}$$

rise to a new species; it is purely a source of progressive change in the same species. The only approach to a double hump which occurs in the curves of human fertility that I have dealt with is a secondary maximum at absolute infertility, due in all probability to artificial restraint on fertility. As those couples who fall into this component leave no offspring, they cannot give rise to a new species.

[5] If there be preferential mating, M_m will not be the mean organ for all individuals. I have adopted the mate mean in order to free the investigations from the influence of this portion of sexual selection.

The first three equations are true, whatever be the distribution of variation in mates, parents, offspring, and fertility ; the fourth equation assumes the standard-deviation of a fraternity or an array of offspring to be $\sigma_0^2(1 - r_0^2)$. This result would flow for normal correlation between organs in parent and offspring, a type of correlation which holds closely for inheritance in the case of man. It would also flow from *any* law of variation which gave a constant coefficient of regression and a constant standard deviation for the array. What, however, is the important point is this, that no assumption has been made with regard to the nature of the fertility correlation. This is essential, as certainly in the case of man this correlation is like the distribution of variation in fertility, markedly skew and not normal in character. Our equations accordingly amply cover facts, which they could not cover had they been solely based on the usual or normal theory of correlation.

(3) By simply forming the means for any organ (or characteristic) for mates and for parents, we can ascertain from Equation (i), if there is or is not any sensible correlation between that organ (or characteristic) and fertility. Equation (ii) enables us to verify the value found for ρ, since σ_p and σ_m are easily calculated when we know the distribution of fertility. If the correlation were normal $S(x^2y')$ would be zero, and this term it may reasonably be expected will never be very large. When ρ has been found from Equation (i), then Equations (iii) and (iv) give us $M_1 - M_0$ and $\sigma_1 - \sigma_0$, or the measures of reproductive selection in its action on the mean and variation of successive generations.

(4) I have applied these results to the only case—that of man—in which statistics are at present available.

I find for upwards of 4,000 families, principally of Anglo-Saxon race, $v = 0.692$, and for 1,842 families of Danish race, $v = 0.652$. This, considering differences of race, is a very satisfactory agreement. In the next place there appears to be a significant difference $0.278''$ between the mean height of mothers of daughters and the mean height of wives. Thus we have $\rho v \sigma_m = 0.278''$, and since $\sigma_m = 2.303''$, it follows that $\rho v = 0.121$. Now, the coefficient of variation for fertility in daughters is not quite the same, but still very nearly the same as that for fertility in general. We therefore find that $\rho = 0.175$ to 0.186, according as we use the first or second value of v given above. We therefore conclude that there is a sensible correlation (*circa* 0.18) between fertility and height in the mothers of daughters.

Turning now to Equations (iii) and (iv), I note that r_0, σ_0, and r_0^2 are multiplied by the small quantities ρ and $1 - (\sigma_p/\sigma_m)^2$, and that r_0 and σ_m only differ from r_p and σ_p by quantities of the order ρ. Hence, neglected to a first approximation ρ^2, we can use the value r_p, already known, for r_0 in (iii) and (iv) and the value σ_1, already known for σ_0 in (iii), we thus deduce—

$$M_1 - M_0 = 0.081''$$
$$\sigma_1 - \sigma_0 = -0.008''.$$

These are the effects of reproductive selection on the height of women. We thus see that the effect is to render women less

variable, and to raise their mean height. The quantities are very small, but it must be remembered that the process is secular. Thus, supposing reproductive selection to have been unchecked by natural selection, say, for forty generations, the mean height of women, neglecting small quantities of the second order, would have been raised about $3\frac{1}{4}$ inches. A factor which would alter stature by about 3 inches in one thousand years is clearly capable of producing very considerable results in the long periods during which evolution may be supposed to have been at work. In the case of both mean and standard deviation the changes from wives to daughters (0·25″ and 0·044″) are, in the only statistics at present available, far more considerable than the above values; but, it must be remembered, that other causes than reproductive selection are at work, such as shrinkage with age and the greater physical training of the young women of to-day.

(5) I have only been able to measure, so far, the actual value of the correlation between fertility and any organ in the case of stature in women. It would, doubtless, be more sensible in other cases, *e.g.*, pelvic measurements. But there are certain considerations which may be referred to here, and which will suggest how important—at any rate in the case of man—it is to take into consideration the influence of reproductive selection.

From considering the fertility of man, in England and in Denmark, I conclude that 25 per cent. of the mated population produce one-half the next generation. This is the gross fertility. Allowing for the selective death-rate—which I knew only for Denmark—27 per cent. of the mated population produced half the next generation. In other words, although natural selection tends to counteract reproductive selection by a death-rate which, it may be shown, rises continuously and uniformly with increased fertility, yet, in the case of civilised man, it is totally ineffectual as against reproductive selection. If we allow for the portion of the population which remains unmarried, we are well within the mark if we say that less than 25 per cent. of one generation produce more than half of the next generation. Correlation, therefore, between fertility and any mental or physical characteristic must work a progressive change.

We know that there are very considerable race and class differences in the matter of fertility. It is very difficult to understand how these could have arisen by the action of natural selection combined with heredity, unless either (1) fertility be inherited, or (2) fertility and some inherited mental or physical characteristic be correlated. But either (1) or (2) involves reproductive selection. We have seen that there is evidence of correlation between the stature of women and their fertility. There is also evidence of a correlation between fertility and class. Taking Copenhagen, for which alone we have satisfactory class-fertility statistics, it is possible to show :—

(i) That the gross fertility of the artisan is more than the gross fertility of the professional classes.

(ii) That the net fertility of the artisan is less than the net fertility of the professional classes.

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Thus natural selection, at first sight, checks reproductive selection, greater fertility connoting a greater death-rate; but we find :—

> (iii) That the marriage-rate of the artisan is so much higher than the marriage-rate of the professional classes, that the percentage fertility of the former considerably exceeds that of the latter.

Thus, while a selective death-rate checks reproductive selection as between class and class, a selective marriage-rate, again, places reproductive selection at an advantage as compared with natural selection; the population would accordingly appear to be ultimately, and in the long run, reproducing itself from the artisan classes.

I hope, later, to publish the analysis, curves, and statistics on which these conclusions are based; at present I only wish to draw attention to the general result: that reproductive selection—at any rate in civilised man—seems a factor of evolution equipotent to natural selection, if, indeed, it be not prepotent.

V.—*British Agriculture in* 1895.

THE annual volume of Agricultural Returns for 1895 recently laid before Parliament (C–8073) embraces the usual information respecting areas and produce of the British crops, numbers of live stock, prices, imports of agricultural produce, together with statistics of foreign and colonial agriculture.

The following passages are summarised from the report on these returns, presented by Major Craigie to the President of the Board of Agriculture:—

Of the entire measured surface of land and water in Great Britain, covering 56,772,000 acres, woods and hill pastures together cover 15,000,000 acres. The cultivated area actually accounted for in the occupiers' schedules in 1895 as under crops or grass, temporary or permanent, is little more than twice this total, or 32,578,000 acres. The area not brought into account under either of these groups includes all the territory occupied for non-agricultural purposes or covered by fences, water, roads, railways, towns, and buildings, as well as the surfaces incapable of profitable occupation, and those held in plots below the acre limit of the holdings whence returns are collected.

The wide variation in the surface character of the areas of the districtive agricultural divisions into which Great Britain is divided for the purpose of the returns is shown, as regards the relative share of territory falling into the several categories just indicated, by the following tabular analysis of the totals, so far as distinguished for 1895 :—