Inheriting Galton's Statistics: George Darwin, Edgeworth and Weldon

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For Karl Pearson, the principal legacy of Francis Galton was the development of simple correlation and multiple correlation. This aspect of Galton's work precipitated Pearson's epistemological shift towards seeing the world in terms of partial causation. It was also through exploring the mathematical properties of Galtonian correlation that Pearson was able to devise a battery of correlational techniques. That the influence extended any further has been successfully challenged by Eileen Magnello. Pearson developed a body of methods in mathematical statistics that lay outside Galton's conceptual framework and beyond his technical scope. He was never Galton's student and he was his intellectual heir in a limited sense only.

Three men may be viewed as having inherited Galton's statistics to a more notable extent. Francis Ysidro Edgeworth and W F R Weldon freely acknowledged their debt in their writings and historians have found no grounds to take issue with their stated positions. Edgeworth shared Galton's belief in the parental gift of intelligence and extended the reasoning to the endowment of high moral fibre and common contentment. He emerged from the milieu of the psychophysicists and psychologists to adopt and develop the Galtonian, atomistic models and normal curve tradition within economics. Weldon, as a young zoologist, was much influenced by Galton's *Natural Inheritance*. In his hands, the domain of the normal curve was broadened to encompass variation in crustaceans, and correlation was applied for the first time outside anthropometry. Weldon also struggled with how to model asymmetrical distributions and prompted Pearson to address the same issues, though in an altogether different way.

Of the third legatee, George Darwin, less has been written. As Charles Darwin's son and Galton's cousin, he was well-versed in the theory of evolution and its importance in the emerging understanding of inheritance. A decade before Edgeworth and two decades before Weldon and Pearson, George Darwin flirted with the Galtonian agenda on inheritance in man and actively supported Galton in the development of regression as a purely statistical concept. His early, albeit unfocussed and diverse, career suggested that he may have had the capacity to both develop the mathematical theory of evolution and design new statistical methods, but under the influence of William Thomson (Lord Kelvin) he was lost to astronomical physics, and it was Pearson who subsequently fulfilled these roles.

1. GEORGE DARWIN AS GALTONIAN

1.1 INTRODUCTION

On 4 January 1875, George Howard Darwin (1845-1912) commented in a letter to his cousin Francis Galton (1822-1911), that we have a 'common family weakness for Statistics'. The letter is actually dated 4 January 1874 but the contents of the letter are such that Darwin must have forgotten the change of New Year.[1] Section 1 will focus on the assistance that George Darwin gave to Galton in the mid-1870s at a time when Galton was unveiling the method of ranks and the concept of regression.

Galton was born into a family of industrialists and bankers in 1822. He initially seemed destined for a career in medicine and was apprenticed at sixteen to a Birmingham hospital. He moved to London in October 1839 to continue his medical education at King's College and the following year took the second prize in anatomy and chemistry. Soon after, Charles Darwin persuaded Galton to 'read Mathematics like a house on fire', and so in 1840, he went up to Trinity College, Cambridge to do just that.[2] Such was his early potential at Cambridge that Galton was quickly selected by the celebrated coach, William Hopkins, for the intensive training programme intended to lead to a lofty position among the wranglers in the Mathematical Tripos.[3] He initially found the tutelage exhilarating and reported to his father that 'I never enjoyed anything so much before'.[4] But as the pressure increased he buckled, a complete breakdown ensued and he quit Cambridge, having taken only the examinations for a poll degree in January 1844.[5]

Galton's early scientific career was shaped by his travels to distant parts funded by his family. His first tour extended through Europe and on to Egypt and Syria. During a subsequent major journey of exploration to south-west Africa he developed into a geographer, meteorologist and ethnologist with an inclination for supplementing his travelogues with statistical information, and as such he entered the higher echelons of the scientific establishment.

By the time Galton returned from Africa, in 1852, he was convinced that intelligence was inherited and that its distribution differed from race to race. At this time, he had no theory of descent but, as this began to emerge, one of its focuses would be artificial selection, rather than natural selection already appreciated by his cousin, Charles Darwin. Galton was conscious of Quetelet's adoption of the normal curve as a model for the distribution of physical characters. Although this awareness possibly arose from the error theory applied to the calibration of instruments at the Kew Observatory and through his contact with James Glaisher, John Herschel and George Airy, Galton later attributed his knowledge of the curve to William Spottiswoode.[6] By 1865 Galton was asserting that both physical and 'psychical' characters were passed on from father to son, and in *Hereditary Genius*, published in 1869, he argued by analogy that intelligence was distributed through the population according to the normal law.[7] He believed that normal curves with different means but equal dispersions described well both the distribution of abilities of different races at any moment in time and the abilities of a single race at different points in time. In a pessimistic paper of 1873, 'Hereditary Improvement', Galton blamed the 'mischievous influences of artificial selection', including differential birth rates, for the deterioration in mental and physical attributes in the great nations of the world, Britain included.[8]

Having recommended to Galton that he read mathematics at Cambridge, Charles Darwin offered the same advice to his second son, George Darwin. The young Darwin was coached by Edward Routh and much to the family's surprise, graduated as Second Wrangler in 1868 just as Galton was writing *Hereditary Genius*. Galton and Charles Darwin were grandsons of the twice-married, Erasmus Darwin, via two distinct lines. George Darwin was born in 1845 and consequently by the time he came down from Cambridge, his cousin, Galton was already 46 years old (see Figure 8.1). Like Galton before him, Darwin was unsure which career to follow. He initially trained as a lawyer but by 1872 he was back at Trinity College Cambridge as a Fellow and exploring a number of intellectual avenues.

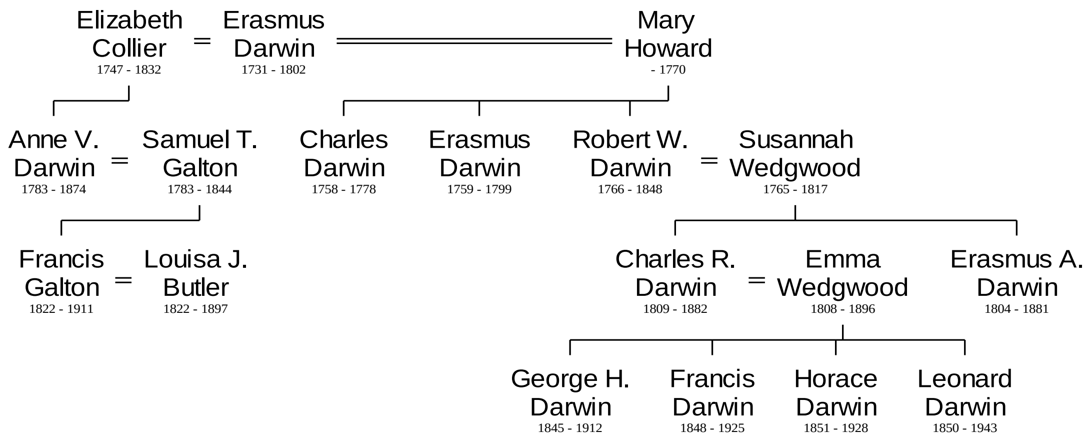

Figure 8.1: The descendants of Erasmus Darwin

Initially, Galton and George Darwin met only infrequently at family gatherings, but theirs was to become an intellectual alliance following the British Association meeting at Liverpool in 1870, at which Galton read a paper on the effects of the weather on the speed of sailing ships.[9] The paper prompted a correspondence through which solutions to problems were sought, reached and refined, and from which it is clear that Galton, far from developing statistical methods in a vacuum in the 1870s, was receiving considerable assistance. Suddenly we find numerous records of visits by Darwin to Galton's Knightsbridge home.[10] Under Galton's influence, Darwin began to take an interest in geography and meteorology, paralleling Galton's youthful enthusiasm for subjects associated with travel. By 1873 at the latest, he turned his attention to the construction of topographical models and large-scale globes, perhaps the most mathematical aspect of geography.[11] But he also began undertaking research into statistics, heredity and eugenics, so much so that if anyone could have been called a 'young Galtonian', at any point in Francis Galton's long career, it was George Darwin in the middle years of the 1870s.

1.2 GEORGE DARWIN'S DEVELOPMENT AS A STATISTICIAN

Darwin shared Galton's views of society, including the desirability of seeking ways of improving it, and indeed, his younger brother, Leonard, would later become one of the leading eugenicists in Britain.[12] In 1873, George Darwin wrote a paper on the desirability of restricting the right of the individual to choose a marriage partner.[13] Both he and Galton were concerned that differential birth rates across the strata of Victorian society adversely affected the general intellectual level of the nation. However, when it came to considering a solution to the problem, they attacked it from opposite ends. Galton suggested that without coercion, men and women who were aware of their natural talents would choose to marry within their cast and to begin their families early.[14] Darwin promoted the view that 'inferior members of the race' be prevented from having children. He favoured not the contrived relative fecundity of the social (and hence intellectual) élite but the contrived relative sterility of the lowly.[15]

Also in 1873, Galton formulated a problem on the probability of the extinction of families, to which a rather unwieldy solution was suggested by Henry William Watson (1827-1903). This has passed into the statistical literature as a seminal paper on branching processes.[16] It is a measure of Galton's increasing confidence in Darwin that he wrote to him, asking 'Is it really hopeless to obtain a more manageable solution?[17]

Later the same year, Charles Darwin wrote to *Nature* to explain how under the theory of evolution the normal curve of physical characters would lose its symmetry as species separate.[18] George Darwin clarified his father's notions in this regard and provided the journal with a second letter, 'Variations of Organs', in which he argued that the symmetry would be restored and, thereafter, the changes in the distribution would proceed in a cyclical pattern.[19] For a detailed account, see Chapter 5 of this volume.

Figure 8.2: George Howard Darwin

Darwin was beginning to get a feel for statistical arguments and over the winter of 1873-74 he explored the mathematics of the normal law, with a view to enhancing his studies in eugenics. At a meeting of the (London) Mathematical Society on 9 April 1874, he read a paper 'On Probable Error in Statistics', in which he sought to establish the distributions of products and quotients of normally distributed variables.[20] This paper never came to publication because J. W. L. Glaisher (1848-1928), who was in the audience, drew Darwin's attention to that fact that the results had already been discovered by Pierre Simon Laplace (1749 – 1827) and restated by Augustus De Morgan (1806-1871).

Meanwhile, Darwin had embarked on a statistical treatment of an aspect of inheritance, the incidence and effects of marriages between cousins. In a paper published in the early part of 1875, Darwin estimated the proportion of first cousin marriages to all marriages at between 3.2 % and 3.9% and perhaps as high as 4.5% among the upper classes.[21] He used the 1851 census figures and other sources, including the returns from a questionnaire, which at the time was also being pioneered as a vehicle for data collection by his father and by Galton. George Darwin found no evidence of a link between first cousin marriages and either a reduction in ability or deficiencies of the senses, much to his father's relief, as Charles had married his own cousin, Emma Wedgwood and three of George Darwin's siblings had perished young.[22] Galton simply commented that the conclusions reached would have had more authority if tempered by probability statements.

1.3 GEORGE DARWIN AS A CONDUIT TO JEVONS AND GLAISHER

Whilst Darwin was developing into a statistician in the Galtonian mould, Galton himself had devised a radical new statistical method by which the magnitudes of characters that are not directly measurable could be ascertained. Physical characters, such as arm length, can be measured directly: there is a metric, a linearly graduated scale against which comparison can be made. But during the early 1870s, following the partial success of *Hereditary Genius*, Galton wanted to measure intelligence, for which there was no metric.

At a Friday evening discourse at the Royal Institution in February 1874, he explained that:

> At present we are accustomed to deal with averages and the like, which can only be obtained by measuring every individual by a detached standard scale, and going through an arithmetical process afterwards. Now I want to deal with cases for which no external standard exists, and I propose to proceed in quite another way, on the principle that intercomparison suffices to define.[23]

When two quantities are compared, so long as their magnitudes are not very close together, a judgement can be made as to which is the greater. Such a judgement may be referred to as a binary judgement. ‘Intercomparison’ is the ordering of a set of quantities by a series of binary judgements. Galton continued:

> I propose to use a scale founded on the law of the Frequency of Error, which gives a scale of equal parts wherever the law applies, and I use the ‘probable error’ for the unit of the scale.[24]

In brief, a standardised form of the law, termed the ‘common statistical scale’, would be devised, and following the arrangement of the data into an ordered set, key values would be identified. These values would include what was later termed the ‘median’ (the datum dividing the set into a lower and upper subset) and the other ‘quartiles’ (effectively, the medians of the two subsets), and it would be only for these particular values that fine discrimination would be needed. The scheme would benefit from pictorial or graphical representation in the form of an ‘ogive’, which Galton depicted in a paper ‘Statistics by Intercomparison’, the following winter.[25] He termed it a ‘curve of double curvature’, but George Darwin explained to him that ‘curve of contrary reflexure’ would be a more accurate description.[26]

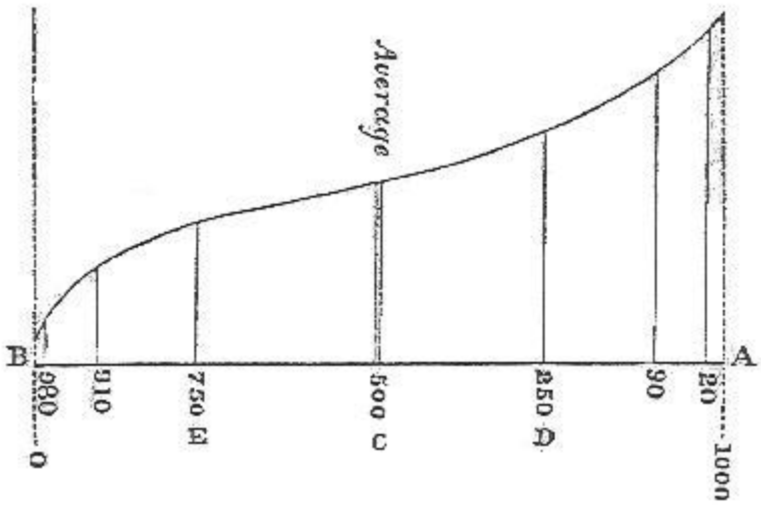

Figure 8.3: Galton’s ogive of January 1875

The question remains as to how Galton came to the idea of attaching magnitudes to psychical characters using a non-linear scale graduated, via the normal curve, in multiples of the probable error. The answer lies in the psychology of Alexander Bain (1818-1903), as adopted for economics by William Stanley Jevons (1835-1882) and intimated to Galton by George Darwin.[27] During the same period that Darwin was writing his statistical papers, he was drawn towards economics, attracted by its sudden redirection towards mathematics in Jevons’ book of 1871, *The Theory of Political Economy*.[28] Jevons based his theory on hedonics, a branch of philosophy that treats of pleasure.[29] He began with a consideration of the intensity and duration of pain and pleasure, the amount of either being the product of these two variables. Expressing the intensity as a function of time, this amount is given by the area below a curve and hence measured by integration. Then, defining a commodity as something that affords pleasure or pain, the utility of a commodity is the area below the commodity-intensity curve, by analogy with the previous mathematical analysis of hedonism.

Jevons' revolutionary approach challenged the received theories, and accordingly elicited many responses. It prompted a supportive response from George Darwin in 1873. In 'Commodities versus Labour', Darwin voiced approval for Jevons, indirectly, by leading an attack on John Stuart Mill.[30] In November of the following year, Jevons urged Darwin to defend the theory against the criticism of another influential economist, John Elliott Cairnes who had completely failed to grasp the mathematics of the new theory.[31] Darwin detailed the mathematical points in Jevons' arguments that Cairnes had misunderstood and went on to develop the theory further, arguing that utility is a function of quantity, intensity and duration, and in the process, persuading Alfred Marshall to accept Jevons' theory.[32]

The influences on Jevons at this time included the philosophies of Jeremy Bentham and Alexander Bain, the latter in particular. Bain conceived a threefold division of mind into intellect, feeling and will, the last two of which he treated in his book *The Emotions and the Will*, published in 1859 and considered a standard text for students of the emerging discipline of psychology for a generation.[33]

The challenge of measuring the unmeasurable was clearly laid out in the preface and first chapter of Jevons' book. Jevons began by explaining why it was that he was making an association between economics and hedonics:

> I have attempted to treat Economy as a Calculus of Pleasure and Pain, and have sketched out, almost irrespective of previous opinions, the form which the science, as it seems to me, must ultimately take. I have long thought that as it deals throughout with quantities, it must be a mathematical science if not in language.[34]

But a science is not mathematical unless it deals with quantities which are measurable, and therein lies a major difficulty, for

> We cannot weight, or gauge, or test the feelings of the mind; there is no unit of labour, or suffering, or enjoyment. It might thus seem as if a mathematical theory of Political Economy would be necessarily deprived for ever of any numerical data.[35]

There is a singular precedent, Jevons noted:

> Previous to the time of Pascal, who would have thought of measuring *doubt* and *belief*? Who would have conceived that the investigation of petty games of chance would have led to the creation of perhaps the most sublime and perfect branch of mathematical science — the theory of probabilities?[36]

Jevons believed there to be a direct relationship between our psychological state and our propensity to engage in economic activity, so that the magnitude of economic activity, which is ascertainable, can give a measure of our psychological state:

> Far be it from me to say that we ever shall have the means of measuring directly the feeling of the human heart. A unit of pleasure or of pain is difficult even to conceive; but it is the amount of these feelings which is continually prompting us to buying and selling, borrowing and lending, labouring and resting, producing and consuming; and it is from the quantitative effects of the feelings that we must estimate their comparative amounts.[37]

He went on:

> Many readers may … consider it quite impossible to create such a calculus as is here contemplated, because we have no means of defining and measuring quantities of feeling, like we can measure a mile, or a right angle,

> or any other physical quantity. But we only employ units of measurement in other things to facilitate the comparison of quantities; and if we can compare the quantities directly, we do not need the units.[38]

Jevons was arguing that the strength of characters which are nor measurable directly can be estimated from the magnitude of some resulting action by filtering the data through a mathematical function, and that the magnitude of the resulting action need only be expressed in an ordinal form in the first place. Here we have all the key components of Galton's method of ranks and use of the ogive.

There is reason to believe that as Galton began thinking about measuring intelligence and other 'psychical characters', he was also dwelling on the words of Jevons: 'the ultimate quantities which we treat in Economics are Pleasures and Pains, and our most difficult task will be to express their dimensions correctly'.[39] If this is so, then the first beneficiary of Jevon's hedonimetry was not Edgeworth as previously thought, but Galton via George Darwin.

This was not the only way in which George Darwin was to play a crucial linking role, for through his friendship with James Whitbread Lee Glaisher (1848-1928), he was able to help Galton understand some of the mathematics of the ogive. The role of Glaisher in the early history of the curve is explored in detail in Chapter 11 of this volume and a brief synopsis is provided here.

Educated at Trinity College Cambridge, Glaisher graduated as Second Wrangler in 1871 and stayed on as a Fellow. He had been lecturing at Cambridge for over two years when George Darwin returned there in October 1873. Glaisher would develop into a mathematician of huge value to his colleagues as a journal editor. His own area of expertise lay in error theory and here he had few peers. In the early 1870s he identified certain integrals with the error function and its complement, and he wrote widely and authoritatively on the mathematics of these functions, tables of their values and their history.

Glaisher and Galton had never met prior to a felicitous encounter late in 1874 when Galton was working on his 'Intercomparison' paper. By this time, Glaisher and Darwin were colleagues at Trinity College and already good friends. The encounter was brief, though long enough to leave Galton with a most favourable impression. 'It gave me great pleasure to make a hurried acquaintance with your winning friend Glaisher', he reported to Darwin.[40] When Galton and Darwin met soon after, the ogive was certainly discussed, and it was a theme of the letters they subsequently exchanged. Over the Christmas period, they clarified which was the independent and which the dependent variable in Galton's ogive.[41] And it appears that Galton requested that Darwin supply him with a formula for the curve because that is what he certainly attempted to do in the very letter of 4 January 1875 which alluded to the 'common family weakness for Statistics'. It is unlikely that Darwin had been able to consult Glaisher directly in those days around the New Year, when Trinity College was not in session, but he was aware of Glaisher's papers and he provided Galton with a gamma function, adding that he only thought he had the correct form.[42] Twelve months later, upon Galton's prompting, Glaisher supplied a full explanation, incorporating the error function and its complement and this he shared with Darwin.[43] But a plan for Glaisher to publish new tables for the ogive never came to fruition.

1.4 PREPARING FOR THE ROYAL INSTITUTION LECTURE ON REGRESSION

In the autumn of 1875, Galton undertook substantial experiments on inheritance in plants. Galton's sweet pea trials were aimed at learning something of the degree in which characters are passed on from one generation to the next. He was aware that inheritance in other flora and particularly in animal species is often more complex than in sweet peas, but he had every intention of applying any worthwhile results as widely as he could. The experiments proved an opportunity to confirm his physiological theory of heredity, especially its major consequence, the stability of types, and to quantify the propensity of offspring to revert to that type. Though it was not his intention, Galton took the first steps towards developing regression analysis, statistical methods that are valuable in a much wider context.

Galton decided to embark on a large-scale experiment. From thousands of seeds he selected those with weights closest to 0, ±1, ±2 and ±3 units on his common statistical scale. The filial seeds were allocated to these seven classes, according to the weights of their parent seeds. Galton persuaded friends in various parts of Britain from Cornwall to the Moray Firth, to plant a set of the seeds for him and report the progress of the progeny. The friend in Kent was Charles Darwin, George having 'volunteered' his father as early as April that year.

The original data are not available, though in 1886 Galton did give a summary table of seed diameters, rather than weights, in his paper on 'Regression Towards Mediocrity in Hereditary Stature'.[44] Galton argued unconvincingly that it was perfectly justifiable to take the diameters rather than the weights, because he had found the two measures to be in direct proportion. The weights in each of the filial groups proved to be distributed according to the normal law. Furthermore, not only was the probable error the same for every filial group but, to Galton's initial surprise, it coincided with the probable error of the weights of the parental seeds. But if the dispersion of weights of parental and filial seeds were equal, their means were not. They were found by Galton to lie somewhere between the parental mean weight and the population mean weight. The average deviation in the weights of the filial seeds from the population mean weight was directly proportional to the average deviation in the weights of the parental seeds from the population mean weight. The constant of proportionality, moreover, was measurable. This propensity for seed weights to adjust towards the population mean, which Galton referred to as 'reversion', countered the propensity for them to become more variable, generation on generation, guaranteeing stability. Galton had reached the two main results; namely, the linearity of regression and the equivariability of the distributions of offspring weights.

But when it came to explaining his sweet peas findings, Galton had a major problem and leant heavily on George Darwin when writing them up for publication. A first draft was completed by May 1876, given a provisional title, and dispatched to George Darwin for his views. Galton wrote:

> May I take the great liberty of asking you kindly to look at a short paper I have just finished concerning peas. 'Experiments with plants, on the causes of statistical uniformity in successive generations'. I should be truly glad of criticism before I send it in, while there is still opportunity to alter. I want to be (1) correct (2) intelligible.[45]

The fact that this first attempt to write up his conclusions is no longer extant, that it took eight months for a second draft to be produced and that the new paper did not retain the original title, suggests that the criticism it received from Darwin was so severe that Galton discarded it altogether and simply began again. And this is confirmed by his next letter, written a week later:

> How can I thank you sufficiently for the great trouble you have taken about my paper, and your criticism. I will wholly, or almost wholly rewrite and expand, and not think of sending it in, in its present form.[46]

The problem may have lain in conveying the characteristics of a law of error compounded of other such laws. A rather frustrated Galton wrote:

> These confounded law of error ideas, which in themselves are so simple & clear but to express which no proper language exists, and which lie so completely out the every day lines of thought, are very baffling to deal with and to present. But I don't despair yet. However I won't bother you more until I have had another good drive at the thing, and then, as you kindly permit me, I will ask you about it.[47]

The second draft completed at the end of the year was effectively a different paper and Galton again sought Darwin’s approval:

> May I venture to trouble you with a request — not a great one. It is to look through a short clearly written (orthographically I mean) memoir on ‘Typical laws of Descent’ which I propose sending to the R[oyal] Soc[iety] & wh[ich] w[oul]d occupy 4 or 5 pages of the Proceedings — and tell me if it is sufficiently intelligible.
>
> You did me real good service in burking my memoir of last year. This is certainly much better than that, but tell me — is it good enough? I will send it at once, if you will have it.[48]

The tone of the postscript suggests that Galton was much more confident about the new paper, appearing unworried about the possibility of another full rewriting.

> Pencil anything you like in it. If possible, I want to send it in *soon* to the Royal Soc[iety], so as to be read before my Feb[ruary] 9 lecture.[49]

The correspondence suggests that he was quickly informed that his confidence was misplaced. With an increase in urgency, both George Darwin and his father wrote to Galton. Charles Darwin’s letter contained an invitation to lunch on the following Sunday (9 January) so that George and Galton could work together on a further redraft.[50] The letter was written from Bryanston Street in Marylebone which, for Galton coming from Rutland Gate, would have been a stroll through Hyde Park to the luncheon appointment.

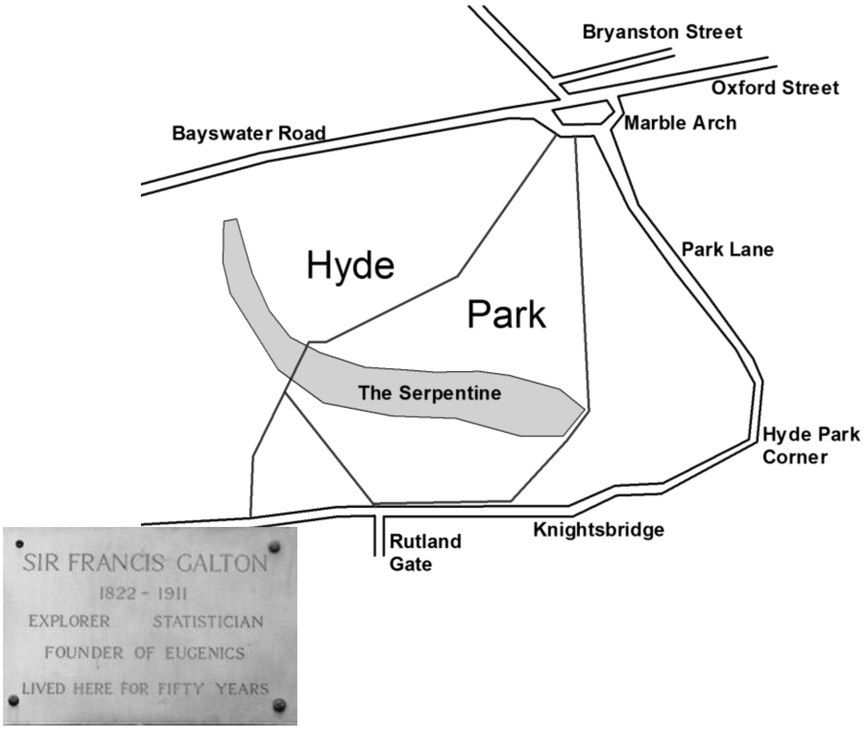

Figure 8.4: Plaque on 42 Rutland Gate and the route to the Darwins’ London residence

The content of George’s letter remains unknown but in a reply of Friday 12 January, Galton assured him that he would have a redraft ready for their meeting:

> How can I thank you sufficiently. I am aghast at the trouble my unlucky memoir gives, and at the great pains you have taken to put clearness into it. I will certainly adopt your suggestions generally, & rewrite the thing.[51]

He also wrote on the same day to let Charles Darwin know of George’s efforts on his behalf:

> I will try hard to put in practice your valuable hints about making my lecture as little unintelligible and dull as may be and have hopes of succeeding somewhat. George has *most kindly* taken infinite pains to the same end.[52]

So it was the second draft of the second paper that finally received George Darwin's valued approval.

On 9 February 1877, Galton delivered a Friday evening lecture at the Royal Institution on 'Typical Laws of Heredity'.[53] As we have seen, the talk had been redrafted following suggestions received from George Darwin and, to lesser extent, Charles Darwin. The lecture's importance in the history of statistics in biology is possibly without parallel for here Galton explained, for the first time, how the processes of variation and reversion to type are both governed by the normal law and how they counter each other to produce stability. The justification for Galton's assertions was presented in an informal way using his pedagogical and heuristic device, later termed the quincunx, and the difficult mathematics held back and presented later in appendices to the written record of the event.

Galton had commissioned the company of Tisley and Spiller to construct the quincunx over the winter of 1873-74 and so had been in possession of it for about three years. Shortly before the talk he realised that it would require an adaptation if it were to be used to explain his new insight. He conveyed his ideas in a famous letter of 12 January to George Darwin, just four weeks before the engagement.[54] It is largely as the recipient of this letter, a wholly passive role, that George Darwin's contribution to the development of Galtonian statistics has been recognised hitherto. Tellingly, there is no reference in the letter to a new two-stage quincunx either having been constructed or to plans for its construction. Tisley and Spiller would have needed far more than a month to build this complex device to the levels of precision required to produce convincing empirical results in a live demonstration at the Royal Institution. The two-stage quincunx was simply the central device in what would now be termed a 'thought experiment' and the idea appears to have come to Galton in the three days between the emergency conference at the Darwins' residence in Marylebone on 9 January and Galton's letter of the twelfth.

'Typical Laws of Heredity' contains a lithograph of this two-stage quincunx, with the upper stage to illustrate the effect of reversion to type and the other to illustrate the effect of variation. On the top landing, the shot represent the normally distributed weights of the parental seeds in the population as a whole. Allowed to pass to the middle landing, they are channelled in such a way that they take closer order. Galton drew attention to the failure of the extreme characters to be sustained generation on generation. When conditions were unfavourable, he argued, adults possessing characters significantly in abundance or deficiency are less likely to survive to pass on their characters. Those that do survive produce fewer offspring of the species and the characters of the offspring that are produced are subject to reversion to mediocrity. But the phenomenon of reversion persists even under favourable conditions of existence and is fabricated in the quincunx by the slope of the channels in the upper part.

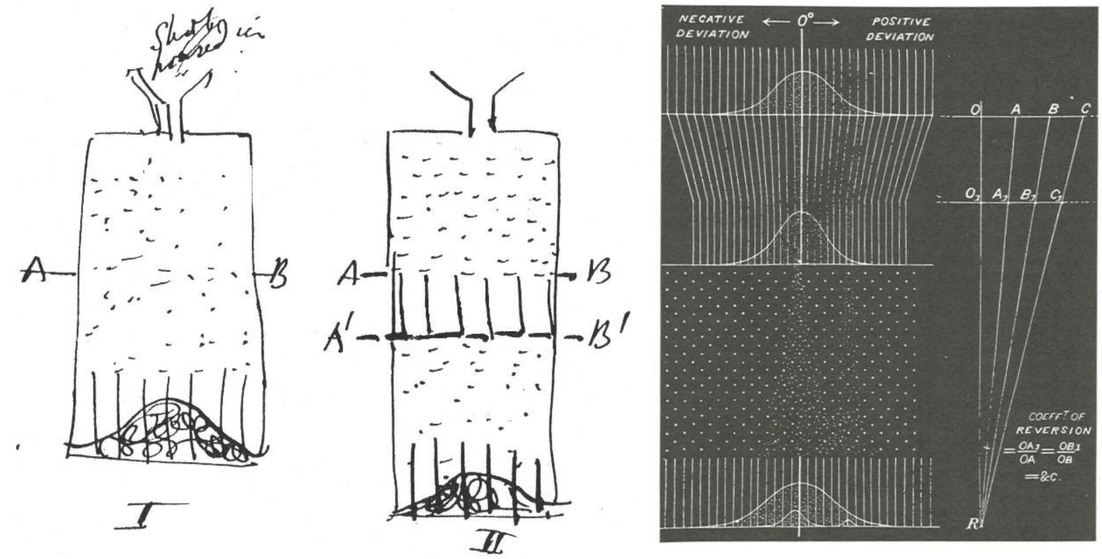

Figure 8.5: Galton's two-Stage quincunx as drawn by Galton in his letter to George Darwin and as depicted in 'Typical Laws of heredity'

Once all the trap-doors on the middle landing are opened the individual heaps merge to form one large array on the bottom landing. The central compartments contain larger 'piles' than the more marginal compartments because they contained more pellets on the middle landing. This represents the distribution of the filial weights, its form and parameters as they were for the population on the top landing. The process of family variation, dummied by this second stage, leads to a greater dispersion. Reversion and family variation are seen to counteract each other, whilst working in harmony with the normal law.

Francis Galton and George Darwin had both a 'common family weakness' and an uncommon family strength for statistics. Galton had such a feel for the subject that he knew instinctively in what direction it needed to be developed. He also had a skill for finding the right vehicle for conveying difficult new concepts, the quincunx being a classic example. Darwin had a better understanding of the mathematics underpinning statistics and could identify the strengths and overcome the weaknesses in an argument. And he could open up new avenues to Galton through his contacts with other Cambridge or Cambridge-educated mathematicians. Taken together with his unstinting and scrupulous efforts on Galton's behalf, there is substantial evidence that George Darwin was an important influence on the development of statistical methods in the 1870s. Without his expertise and support the full panoply of Galtonian statistics would not have been brought to public notice in the form and at the time that it was.

2. EDGEWORTH AS GALTONIAN

2.1 INTRODUCTION

In the period immediately following George Darwin's loss to astronomy, Galton's focus was on extending the domain of the law of frequency to aspects of psychology other than intelligence.[55] His 'Statistics of mental imagery' appeared in James Sully's journal, *Mind* in July 1880.[56] It contained the method of ranks, being used 'for the first time in dealing with psychological data'.[57] Galton identified the quartiles, the first and last octiles and the first and last suboctiles and he included the skeleton of an ogive, the station ordinals standing proud but without their apices connected.

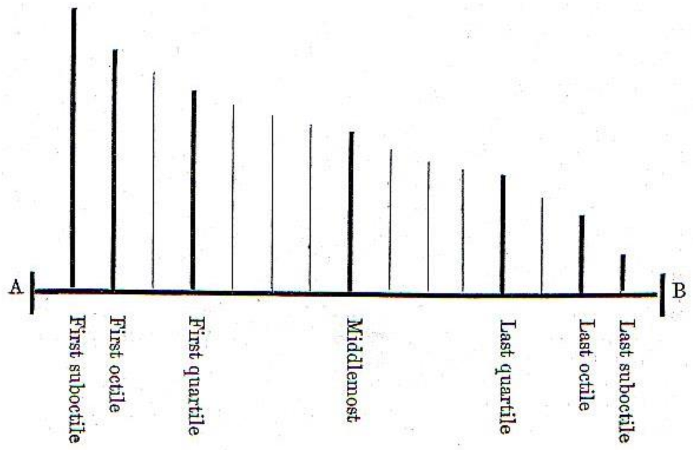

Figure 8.6: A skeleton ogive in Galton's first psychometric paper

Another paper on 'Mental imagery' appeared in the *Fortnightly Review* in September 1880.[58] It is a rather incomplete record of a popular lecture given by Galton at the Swansea meeting of the British Association the previous month.[59] The audience was given the opportunity to learn directly about the method of ranks and the use of the ogive, but they would have noticed also that the standard shape of the ogive was somewhat missing as the distribution of skill in visualizing was only approximately symmetrical.

Already in the paper 'Statistics of mental imagery' Galton had argued that the object of statistics is to '*discover methods* of epitomising a great, even an infinite, amount of variation in compact form'.[60] The transition from error theory to variation theory was completed, however, with the writing of an anthropometric and auxological report for the British Association in 1881.[61] Here the probable error was completely subsumed by the interquartile and interdecile ranges.

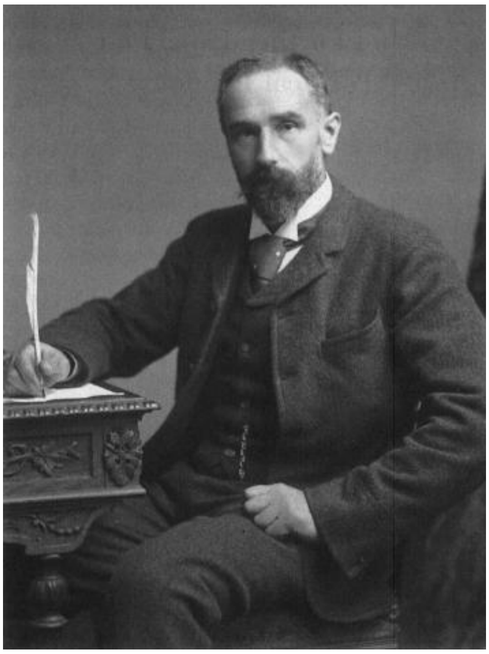

Figure 8.7: Edgeworth

Enter another acolyte, Francis Ysidro Edgeworth (1845-1926), the same age as George Darwin, and equally as slow at getting his career going. And enter another relative, though this time, not a blood relative. Edgeworth's cousin Harriet Jessie Edgeworth married Arthur Gray Butler, whose sister, Louisa, married Francis Galton, as can be seen from part of the Edgeworth-Butler family tree:

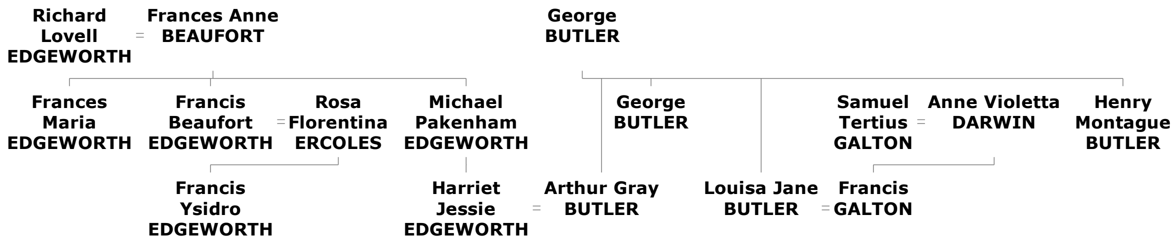

Figure 8.8: The Edgeworth-Butler family tree

Edgeworth sought to extend the approach of the psychophysicists, the mathematics of the error theorists and ultimately the statistics of Galton to the social sciences and economics. Galton recognised his potential for research in the area towards which Galton's own programme had recently led him and certainly, he would have welcomed a co-worker. But if collaboration was on Galton's mind in the 1880s, it appears that it was unrequited. Their correspondence dates in the main from the end of that decade and even at that time, Edgeworth was writing (in relation to correlation) that, 'I am sorry that I am not just at present free to enter more fully into the new and original methods which you struck out'.[62] The flow between Galton and Darwin was two-way, but between Galton it flowed almost exclusively towards Edgeworth. Stephen Stigler wrote that the two men 'never developed a close working relationship', but that 'it is reasonable to surmise that conversations with Galton helped to intensify Edgeworth's interest in statistics in the early 1880s'. And he judged of Edgeworth's statistical publications that 'much of this work bore the stamp of Galton's influence'.[63] These are conclusions that are supported by the following evidence.

2.2 HAMPSTEAD, HOME OF PSYCHOLOGISTS AND ECONOMISTS

After leaving Oxford, Edgeworth lived in Hampstead with little financial support. Amongst his neighbours was Jevons who befriended him. Jevons lived in Branch Hill, Edgeworth in Mount Vernon nearby, and the two often walked together on Hampstead Heath. James Sully moved to East Heath Road, Hampstead in 1878 but he was 'probably Edgeworth's closest friend' for some time before that.[64] Indeed, Edgeworth provided Sully with comments on a draft of *Pessimism*, published in 1877. Sully reciprocated by helping to check the proofs of Edgeworth's *New and Old Methods of Ethics* (1877).[65] It appears that it was Sully who introduced Edgeworth to Jevons in 1879, the trigger to Edgeworth's turn towards economics.[66] (Incidentally, Pearson lived at 7 Well Road, on the edge of the heath.)

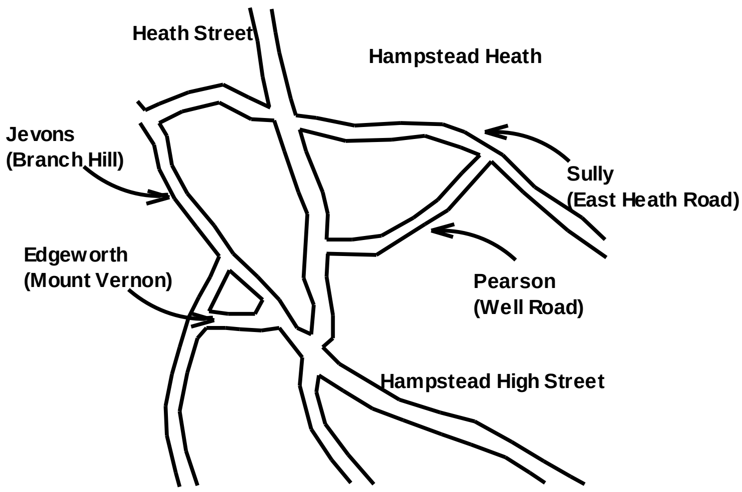

Figure 8.9: Hampstead's fraternity of psychologists and economists

Edgeworth was a tutor and friend of Philip Wicksteed (1844-1927), Jevons's first major follower, who from 1882 onwards developed Jevonian economics in a non-statistical direction and who corresponded extensively with Pearson from 1885.[67]

2.3 BELIEF IN INNATE SUPERIORITY

In his first book (of 1877) *New and Old Methods of Ethics*, Edgeworth displayed an unapologetic belief in inherited superiority. He wrote:

> Unto him that hath higher development shall be added more of the world's goods. This deduction agrees with common sense, as exhibited in the approved dealings of men with animals, of civilized with savage races, in the privileges of aristocracy approved in ages when aristocracies really represent a higher order of evolution.[68]

Galton's whole research programme had stemmed from an unquestioning belief and desire to demonstrate that intelligence is inherited. When Edgeworth came to consider the inheritance of morals and the inheritance of pleasure or happiness, his Galtonian leanings were much in evidence. On morals he wrote, 'Common-Sense morality is really only adapted for ordinary men in ordinary circumstances', adding that there is a need to:

> ascertain how far people in special circumstances require a morality more specially adapted to them than Common Sense is willing to concede: and also how far men of peculiar physical and mental constitution ought to be exempted from ordinary rules …[69]

He observed with regard to pleasure:

> A sentient is said to have greater felicific power when he not only obtains a greater quantity of pleasure … with the same quantity of material means … but also a greater increment of pleasure from the same increment of means … Height of evolution is probably a mark of 'felicific power'.[70]

The influence of those who possessed little ability to experience pleasure or to be happy might be reduced by selective emigration and immigration, by evolutionary pressures or by artificial selection. Citing Galton's *Hereditary Genius* — 'the weak could find a welcome and a refuge in celibate monasteries or sisterhoods' — Edgeworth wrote of 'excluding some sections from a share of domestic pleasures'.[71]

2.4 ATOMISM AND THE MECHANICS OF PSYCHO-ECONOMICS

Reading Charles Darwin's theory of pangenesis upon its publication in 1868, Galton had been struck by the notion of the indivisible transmission unit, the gemmule. He had immediately recognised the possibility of constructing a statistical theory of inheritance from the building blocks of indivisible units, though not Darwin's gemmules. His units of inheritance, termed 'germs', formed a 'stirp' inside the reproductive organs which were subjected to attractive and repulsive forces that effected a complex selection process in accordance with Newtonian physics.[72]

Edgeworth's atomism has been discussed by Theodore Porter, and more recently by Philip Morowski, who gave the title 'Marshaling unruly atoms' to the introductory chapter of his edition of Edgeworth's writings on chance in economics.[73] Edgeworth made great play of the atomic foundations of hedonics in 1881 in *Mathematical Psychics*, and from the outset demonstrated that he understood the advantage provided by the link with probability theory:

> Atoms of pleasure are not easy to distinguish and discern; more continuous than sand, more discrete than liquid; as it were nuclei of the just-perceivable, embedded in circumambient semi-consciousness.

We cannot *count* the golden sands of life; we cannot *number* the 'innumerable smile' of seas of love; but we seem to be capable of observing that there is here a *greater*, there a *less*, multitude of pleasure-units, mass of happiness; and that is enough.[74]

He knew that there was a complexity about the deterministic three-body problem which was singularly missing when a stochastic approach was taken to the behaviour of a vast number of bodies, as had been demonstrated by Fourier and William Thompson in the theory of heat and by Maxwell and Boltzmann with respect to the motion of gases. Edgeworth made this plain in his comment that, 'Mathematics *can* solve the problem of many bodies — not indeed numerically and explicitly, but practically and philosophically, affording approximate measurements …'[75] And he developed the argument in the context of Laplace's celestial physics and Quetelet's extension of it into anthropometry.

"Mécanique Sociale" may one day take her place along with "Mécanique Céleste", throned each upon the double-sided height of one maximum principle, the supreme pinnacle of moral as of physical science. As the movements of each particle, constrained or loose, in a material cosmos are continually subordinated to one maximum sub-total of accumulated energy, so the movements of each soul, whether selfishly isolated or linked sympathetically, may continually be realising the maximum energy of pleasure, the Divine love of the universe.

"Mécanique Sociale", in comparison with her elder sister, is less attractive to the vulgar worshipper in that she is discernible by the eye of faith alone. The statuesque beauty of the one is manifest; but the fairy like features of the other and her fluent form are veiled. But mathematics has long walked by the evidence of things not seen in the world of atoms (the methods whereof, … statistical and rough, may illustrate the possibility of social mathematics). The invisible energy of electricity is grasped by the marvellous methods of Lagrange; the invisible energy of pleasure may admit of a similar handling.[76]

It is interesting that Edgeworth first used a stochastic atomic model in a *reductio ad absurdum* argument in support of utilitarianism.[77]

2.5 STYLE

In the main, Edgeworth's contemporaries were either critical of his style of exposition and expression, or simply nonplussed. And certainly in the early years, much of his writing exhibited an intellectual élitism which was out of place even in Victorian England. It offered a veritable parade of specially-coined words, untranslated quotations from classical literature, an unusual method of labelling paragraphs and allusions galore. Jevons, for example, wrote in his review of *Mathematical Psychics*:

There can be no doubt that in the style of his composition Mr. Edgeworth does not do justice to his matter. His style, if not obscure, is *implicit*, so that the reader is left to puzzle out every important sentence like an enigma.[78]

In recent times, historians have been much more favourable in their comments. Porter dubbed Edgeworth the 'poet of statisticians', while Mirowski thought that he 'deserves to be recognized as … a master prose stylist'.[79] Stigler commented that 'at first such devices as the liberal use … of untranslated phrases or even sentences in Greek or Latin are distracting, but as they become rarer in his later work, we come to enjoy them, as we might some other eccentricity in an old friend'.[80]

Galton initially gained a favourable impression too, and he wrote to Edgeworth to encourage him. The letter, of October 1881, was the first contact between the men (and predates the body of their correspondence held in the Galton Archive by six years).

Permit me to express the very great interest with which I have been reading your powerful work of Math. Psychics, and especially those parts of it that claim the right of Mathematics to deal even with the loosest quantitative data. I write more especially, because I was led to a knowledge of your book by an article by Prof. Jevons in "Mind", in which he happens to speak of its being an unnecessarily difficult book to read. With that verdict I am totally at issue. It strikes me that you have handled topics very difficult in themselves, with great lucidity and vivacity; and I do sincerely hope that you will not suffer yourself to be discouraged by that verdict … It is a grand attempt that you are making … and I trust you will continue to work at various branches of this wide subject …[81]

Of course, Galton would want the support of this emerging talent. But there is a specific aspect of Edgeworth's style that Galton would have found appealing and that is the extravagant use of allusion and analogy, simply because it mirrored Galton's own mode. Edgeworth, no doubt, learnt from Galton's writings of the advantages, perhaps the necessity, of allusion and analogy in psychology and sociology whenever the physicist's atomic model is adopted. In Edgeworth's *Mathematical Psychics* humans (the atoms) were likened to charioteers colliding in a massive amphitheatre. Their close clustering provided strength, as evidenced by the emerging Trades Unions (of which Edgeworth hardly approved):

The one thing from an abstract point of view visible amidst the jumble of catallactic molecules, the jostle of competitive crowds, is that those who form themselves into compact bodies by *combination* do not tend to lose.[82]

Edgeworth's promotion of the normal curve and its associates is considered in the next section, but his allusions are worth quoting at this point. Edgeworth often referred to the error law (now the normal distribution) as having the graphical shape of a 'gend'arme's hat'.[83] By contrast, what we now call a uniform discrete distribution would give the 'outline of a pork-pie hat'.[84] And he extended the analogy to skew distributions, where in seeking to fit the distribution, it was necessary to crumple or stretch the hat to fit the axes. For further details, nicely summarised, see Stigler's 1978 article on Edgeworth.[85] And as another example, when discussing the entries in a two-way table Edgeworth asked his audience to picture:

The site of a city [which] consists of several terraces, produced it may be by the gentler geological agencies. The terraces lie parallel to each other, east and west. They are intersected perpendicularly by ridges which have been produced by igneous displacement.[86]

He went on to describe the positioning of the houses on the terraces and to discuss their mean height above sea level. An allusion can be more powerful than a thousand symbols; Galton knew that and so did Edgeworth.

2.6 PROMOTION OF THE NORMAL CURVE AND THE METHOD OF RANKS

Initially, Edgeworth was well-disposed towards the asymmetrical 'law of facility', described by Galton and made mathematical in the form of what we now call the lognormal distribution by Donald McAlister in papers of 1879 and 1881.

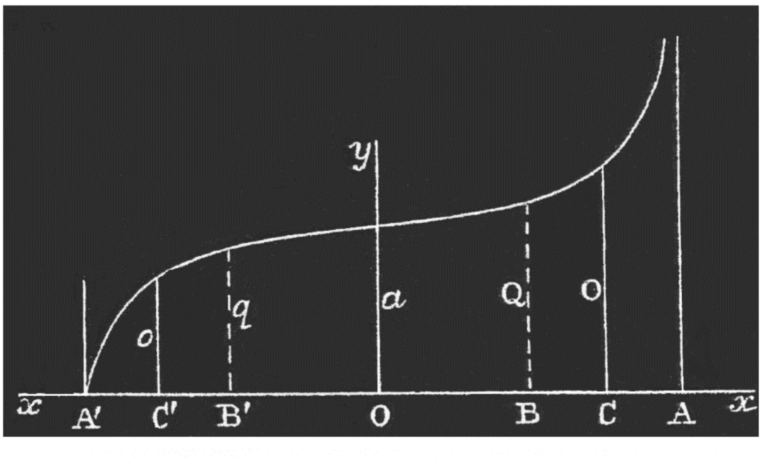

Figure 8.10: Galton-McAlister law of facility (now lognormal distribution)

For Edgeworth, this early focus on the skew distribution arose naturally from his entry into statistics via psychophysics and psychology. *New and Old Methods of Ethics*, contains specific references to variations of Fechner's law which Edgeworth deemed 'applicable to pleasures', and from which the 'conditions favourable to the production of the greatest quantity of pleasure from a given stimulus may be deduced'.[87]

> When we suppose plurality of natures as well as persons, we have to suppose a plurality of contract-curves (which may be approximately conceived as grouped according to a well-known logarithmic law about an average).[88]

When Edgeworth was introduced to Jevons in 1879, the latter had just finished compiling the bibliography for the second edition of his *Theory of Political Economy* and this became Edgeworth's introductory reading list in economics and statistics.[89] Within two years he had assimilated all the latest theory in economics and had begun to produce original work in which he cited economists including Henry Sidgwick (1838-1900), John Elliott Cairnes (1823-1875) and, of course, Jevons.

But by 1883, he was also in total command of the material relating to the foundations of probability and its role in the inductive sciences, discussing with confidence the authorities past and present. Some seven papers were published in 1883 and 1884 alone: 'The Law of Error', 'The Method of Least Squares', 'The Physical Basis of Probability', 'The Philosophy of Chance', 'On the Reduction of Observations', 'A priori Probabilities' and 'Chance and Law'. Here we find references to Laplace, Gauss, Poisson, Herschel, Quetelet, Cournot, Airy, De Morgan, John Stuart Mill, Benjamin Peirce, Donkin, Boole, Renouvier, Leslie Ellis, Crofton, Venn, Charles Sanders Peirce and Glaisher.[90] It is also clear that Edgeworth knows of Fourier's writings on heat and Maxwell on the dynamics of an ideal gas. It is hardly surprising, then, that he had become increasingly enamoured of the law of error. And, indeed, with every passing year from this time on he came to believe more strongly in the universal application of the normal curve.

Galton's influence is seen more directly soon after, notably in 1885, with four outstanding papers which were to mark Edgeworth as a statistician of some gravitas.[91]

- Observations and Statistics
- Methods of Statistics
- On Methods of Ascertaining Variation in the Rate of Births, Deaths, and Marriages
- Progressive Means

These papers have been analysed in great depth, notably by Stephen Stigler, and further details can be found in his books, *History of Statistics* and *Statistics on the Table*.[92] 'Method of Statistics' is Edgeworth's most Galtonian paper, and one which served as a source for the next generation of statisticians, including Weldon and Pearson. Its starting point was Galton's paper 'Statistics by Intercomparison' of 1875 in which there was a recognition that the variability in subpopulations contributes differentially to the variability in the population as a whole. He took as his context the very example that Galton had used, fruit yield from different parts of an orchard and illustrated the argument with a Galton-Darwin diagram of normal distributions with different dispersions combining to produce an overarching normal curve. But it was not the contribution to the population that Edgeworth was concerned with. He simply wanted to seek differences amongst the moduli of the subpopulations as evidence that aspect contributed to the size of fruit.

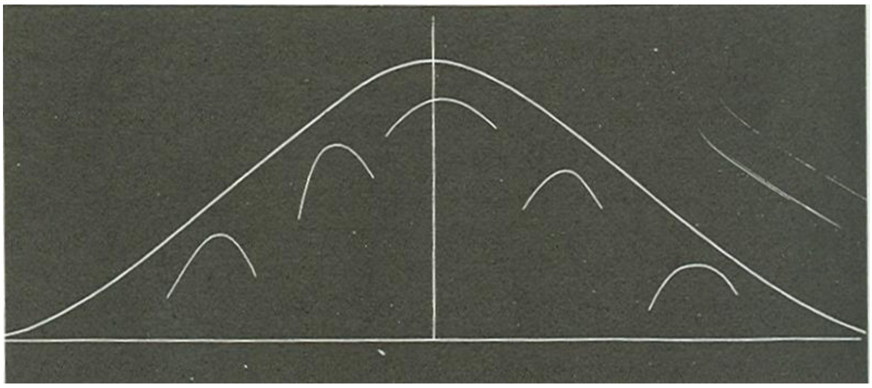

Figure 8.11: Edgeworth's distributions of pear sizes, 1885

In considering the 'law of error', which he gave as $y=\sqrt{C^2\pi}\exp\left[-x^2/C^2\right]$, where C is the 'modulus', he argued that C^2 (now twice the variance) be estimated by $2\sum_{i=1}^{n}(X_i-\bar{X})^2/n$. He was aware of the way in which moduli combined mathematically (both Airy and McAlister had published on the matter) and he used this to devise a test for comparing means or, with adjustment, medians.

The third paper on variation in vital statistics was given at the Aberdeen meeting of the British Association for the Advancement of Science in September, and built on the consideration of the modulus in the previous paper. Edgeworth would have been delighted that Galton was in the audience, just two days after reading his famous paper on 'Regression towards Mediocrity in Human Stature'.[93] In his paper, Edgeworth addressed the problem of how to separate out the confounded effects of different causes (embodied in what he called 'entangled moduli'). Using tabular arrays, rather than a plethora of symbols, to make his talk audience-friendly, Edgeworth's paper presented an early two-way classification, decades ahead of Fisher's analysis of variance.

Edgeworth extended the range of applications of the normal curve and whilst numerous examples might be cited, it might be appropriate to mention the three most significant of them. Firstly, in his paper on 'The Statistics of Examinations' (1888), Edgeworth argued that, with respect to examination marks, 'it is proper to suppose … that they are grouped according to the *law of error*'.[94] He went on to show how scores could be standardised across the markers by rescaling. From any mark, subtract the marker's mean and contract by his semi-interquartile range, stretch by the required semi-interquartile range and add the required mean. Of course, if the supposition of symmetry is incorrect and the distribution of the marks is as a 'hat blown on one side agreeably to the law of Mr Galton and other distinguished statisticians, with respect to the errors or deviations in psychical quantities', then the geometric mean would be the correct central value.[95] The rescaling would require computation on an impracticable scale. (Presumably, rescaling would be undertaken by dividing, taking roots and then powers and finally multiplying.)

In 'The Mathematical Theory of Banking', published in 1890, Edgeworth used the full panoply of the Galton-McAlister terminology of ranks: median, quartiles and octiles, whilst favouring the term 'modulus' (as had McAlister) for the measure of dispersion.[96]

In 1898, Edgeworth showed that the proportion of voters favouring a particular political party was normally distributed.[97] He had analysed the results of the parliamentary elections of 1886, 1892 and 1895. At the first and third of these elections the Conservative (& Unionist) Party was returned under Salisbury and following the 1892 election, Gladstone's Liberals formed the administration (with John Morley, who had been so critical of Galton's methods, as Chief Secretary for Ireland; see Chapter 11). Edgeworth thought it 'evident on inspection that there is an approximation to the normal form', when he considered the ratio U/(U+G), where U and G are the sizes of the Unionist and Gladstonian votes respectively.[98] He then tested the goodness of the fit by comparing the measures of central tendency and dispersion in the error theorist and Galtonian traditions. For each of the three elections, he found an impressive agreement between the means and the medians, and a reasonable agreement between the mean deviations, the mean square deviations and the semi-interquartile ranges, all suitably scaled. Edgeworth concluded that the fit was good except in the tails:

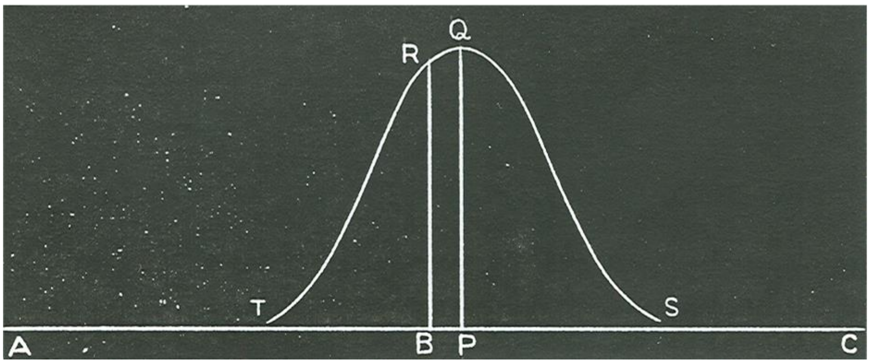

Figure 8.12: Edgeworth's 'normal curve' of voting patterns
[Unionist share of the vote represented by the area to the right of BR]

This interesting 'goodness of fit' procedure would be improved upon almost immediately with Pearson's breakthrough in 1900.

2.7 EDGEWORTH ON CORRELATION

On 10 September 1885, Galton was at the Aberdeen meeting of the British Association for the Advancement of Science, presiding over the proceedings of the Anthropology Section, explaining the latest discoveries to emerge from the measurements made at his Anthropometric Laboratory in South Kensington and addressing the subject of 'family likeness in stature'.[99] Thus began a decade of writings on partial causation, arising firstly through the researches of Galton, in which the concepts of regression and correlation were somewhat conflated, and then from Edgeworth's ideas on correlation. The output (spoken and written) of the two men's work on partial causation is temporally disjointed but spanned by a period of correspondence, indicated by the shading in Table 1,on the following page.[100]

Very briefly, in the papers of 1885/86, Galton studied the inheritance of stature from one generation to the next. His approach included enhancing (or 'transmuting' to use Galton's term) all the data for women by 8% and averaging to form what he termed a 'mid-parent height', so that a comparison could be made with the heights of the offspring. A two-way array of adjusted spot values (which themselves provided the third dimension) revealed numerous patterns. Sections taken in two orthogonal directions each gave a normal curve (the normal distribution of heights of mid-parents and offspring) and sections taken in the third direction gave an ellipse. The visual geometry of Galton's diagram was largely confirmed by the mathematical analysis provided by J D Hamilton Dickson. (The smoothing undertaken by Galton is sometimes taken to have been influenced by his earliest meteorological studies, but the link may be with George Darwin's paper on fallible measures (1877) and Galton's own on a similar subject (1885), in which case, Darwin's reciprocal influence on Galton may have lasted well beyond their immediate collaboration.[101]) Edgeworth was in the audience.

Table 1

Galton	1885	*Presidential Address, Anthropology Section, BAAS*
	1886	Regression towards mediocrity in hereditary stature
		Family likeness in stature
		Family likeness in eye-colour
	1888	Co-relations and their measurement
	1889	*Presidential Address to the Anthropological Institute*
		Natural Inheritance (especially Chapter 7)
	1890	Kinship and correlation
Edgeworth	1892	*Fourth Newmarch Lecture*
		Correlated averages
		The law of error and correlated averages
	1893	A new method of treating correlated averages
		Exercises in the calculation of errors
		Note on the calculation of correlation between organs
		Statistical correlation between social phenomena
	1894	Asymmetrical correlation between social phenomena

Two years later, Galton wrote on 'co-relations', later 'correlation'. His fundamental result was that when two characters are measured, such as stature and cubit, the two regression lines coincide when the measurements are scaled in terms of their unit of dispersion, to Galton the 'probable error'. Stigler has drawn attention to the fact that the complexities of the mathematics of correlation were about to increase and Galton sensed that this was a path he could follow no further.[102] The correspondence with Edgeworth had begun and the direction in which Edgeworth would be taking matters became clear. Indeed he tried to get Galton to assist him by providing data triplets so that he could consider the relationship between pairwise correlation values.[103] Days later he was introducing the expression 'coefficient of correlation' in his Newmarch Lecture, and with quick order he was writing papers on 'correlated averages' in the context of stature, cubit and knee-height triples, in an attempt to extend the domain of correlation to higher dimensions. Not only were these publications in Stigler's words, 'directly inspired by Galton's work', but Galton was providing direct help to Edgeworth as he prepared his material for publication.[104] Incidentally, for the Newmarch Lecture on correlation, Edgeworth, the consummate Galtonian, borrowed Galton's quincunx for his demonstrations.[105]

3. WELDON AS GALTONIAN

Originally destined for a career in medicine, Walter Frank Raphael Weldon (1860-1906) entered University College London in 1876, transferred to Kings College London the following year, and then went up to St John's College Cambridge in April 1878, where he came under the influence of Francis Maitland Balfour.[106] Frank Balfour had had early association with Charles Darwin, attempting to replicate the pangenesis experiments on rabbits. Furthermore, the Darwin boys and the Balfour boys moved in the same social circles. George Darwin played tennis with Arthur Balfour, later Prime Minister, and went shooting with him on his estate in Scotland. Frank Balfour died young, climbing in the Alps, and George Darwin, according to his brother Francis was such a friend that 'George's affection for him never faded', even into old age.[107] In his tribute to Weldon, Pearson referred to Frank Balfour, Huxley and Galton as the three men whose ‘influence tended most to mould his life and career’.[108]

Figure 8.13: Weldon

But the St John’s connection is of interest, not just because of the influential Frank Balfour, but because this was McAlister’s and Pendlebury’s college. Richard Pendlebury (1847-1902) was the Senior Wrangler in 1870, and a famous Alpine climber. It was he who refined J W L Glaisher’s definitions of the error function, erf x, which is the mathematical form of Galton’s ogive, later Pearson’s cumulative frequency curve (see Chapter 11). As we have seen, at Galton’s bidding, McAlister formulated what later became known as the lognormal distribution, with its association with psychology and economics, including the work of Jevons and Edgeworth. McAlister largely turned his back on mathematics to focus on medicine but it was to him that Weldon turned for help when he needed to learn Galton’s methods. In his first paper on shrimp, Weldon recorded his thanks to ‘Dr. Donald Macalister for explaining to me many points connected with the law of error, and for helping me in various ways’.[109] And when, in December 1893, he convened a meeting to establish a committee of the Royal Society to conduct statistical inquiries into biological variation, McAlister was conscripted as mathematician.[110] It appears that McAlister’s contribution to statistics has yet to emerge fully.

Galton and Weldon met for the first time at the Swansea meeting of the British Association for the Advancement of Science in 1880, though it took a further decade for their interests to fully converge and even then they would never agree on the fundamental issue of whether evolution proceeded by small steps or huge leaps. Weldon, and indeed Edgeworth, followed a stricter Darwinian line than Galton.

Weldon wrote a paper on *Crangon vulgaris*, the common shrimp, which was communicated to the Royal Society in March 1890.[111] He acknowledged from the outset that:

> In making this investigation, I have had the great privilege of being constantly advised and helped, in every possible way, by Mr. Galton. My ignorance of statistical methods was so great that, without Mr. Galton’s constant help, given by letter at the expenditure of a very great amount of time and trouble, this paper would never have been written. I am glad to take this opportunity of expressing my gratitude for his generous conduct.[112]

It seems that early drafts had been sent to Galton who was happy to work with Weldon until the paper was both correct and consistent with the Galtonian approach. The voluminous correspondence held in the Galton Archive dates from this period, Weldon writing a letter about every three days.[113] A trawl through this paper and the two that followed on shrimp and crabs, reveals that, with considerable deference, Weldon cited Galton and his statistical methods on no less than 29 occasions.

Weldon sought to extend Galton's studies to 'variations in certain organs in a species living in a wild state, upon which natural selection … may be supposed to act with full effect'.[114] He had recently read Galton's *Natural Inheritance*, published the previous year, and cited from that book the author's prediction that natural selection would not distort the form of the distribution of variation. The results of his study of the common shrimp of Plymouth Sound were 'such as to fully justify Mr. Galton's prediction'.[115]

For the lengths of the carapace in 400 female shrimp, Weldon had proceeded in a purely Galtonian way, ordering the data and constructing an ogive with base line divided into a hundred grades, marking the median M and the quartiles Q_1 and Q_3 and denoting the 'probable error' $Q_1 – Q_3$ as Q. (Note that, contrary to current practice, the upper quartile is Q_1.) He used the term 'normal curve' for the ogive in which the quartiles are symmetrically located about the median. The quality of the fit was established by comparing observed values with the theoretical curve for 13 grades. The median and probable errors of carapace lengths were found to differ for three local races of shrimp, but the distribution of lengths was normal in each case. Similar calculations led to the same conclusion with regard to the lengths of three other organs. Although a preliminary investigation into the correlation between lengths had been undertaken by Weldon, he chose to delay publishing his results until more data were available.

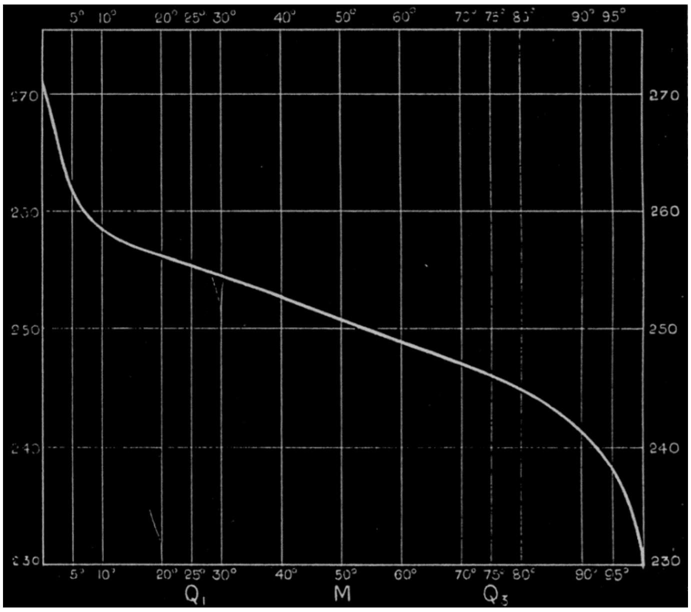

Figure 8.14: Weldon's ogive showing the distribution of carapace length in shrimp (1890)

This came in March 1892, when Weldon published a paper in which the correlation between four parts of the body of the common shrimp was studied.[116] This paper may be considered as the second part of the 1890 paper. Citing Galton's seminal paper on correlation in man, Weldon set out to extend the method to wild species.[117] He established a measure of the interrelation or 'degree of correlation' between two organs in the same individual, and this he did for four local races. In the judgement of Pearson, 'these two papers are epoch-making in the history of the science, afterwards called biometry'.[118]

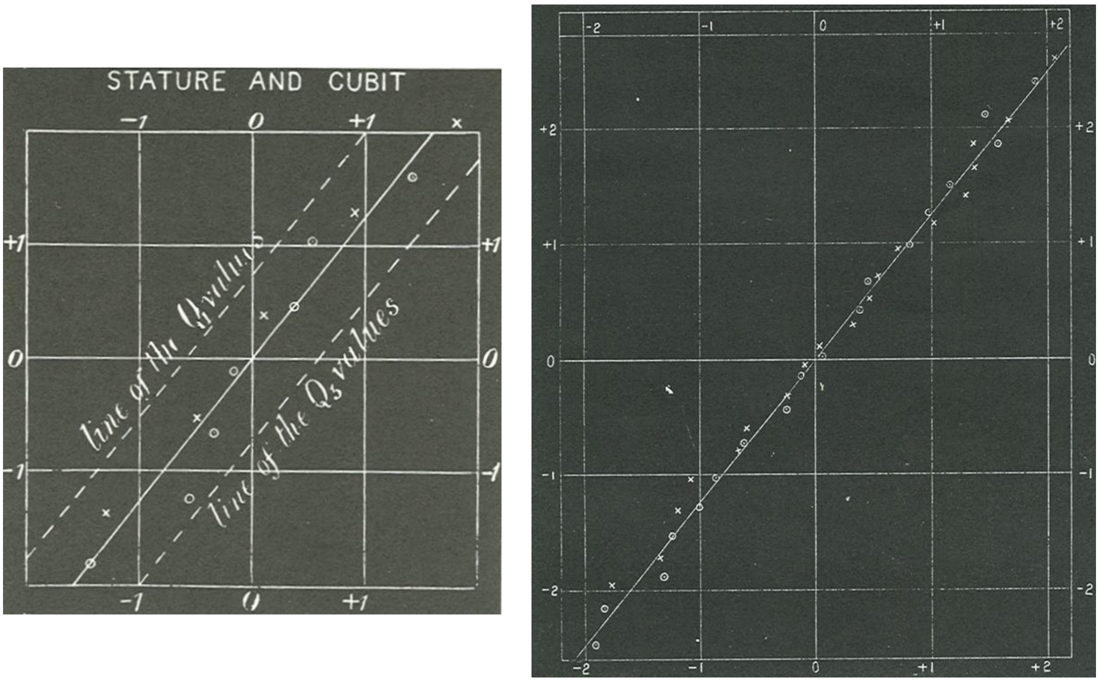

Figure 8.15: Correlation in man (Galton, 1888), and in shrimps (Weldon, 1892)

The correlation studies continued the following year with a paper on shore crabs.[119] The detail is phenomenal, eleven measurements being taken on a thousand female crabs from Plymouth Sound and on a further thousand from the Bay of Naples. Weldon expressed each measurement in terms of the total carapace length, and concluded that the respective distributions of variation were normal in 19 cases (all ten for Plymouth, all but one for Naples), the exception being the distribution of the frontal breadth of the Naples specimens. Weldon thought that there were two local races in the Bay of Naples and Karl Pearson was able to find the parameters of two normal distributions which, taken together, would produce the observed asymmetry. But 'Galton's function' of correlation, as Weldon wished it to be called, failed to support the dimorphism.[120]

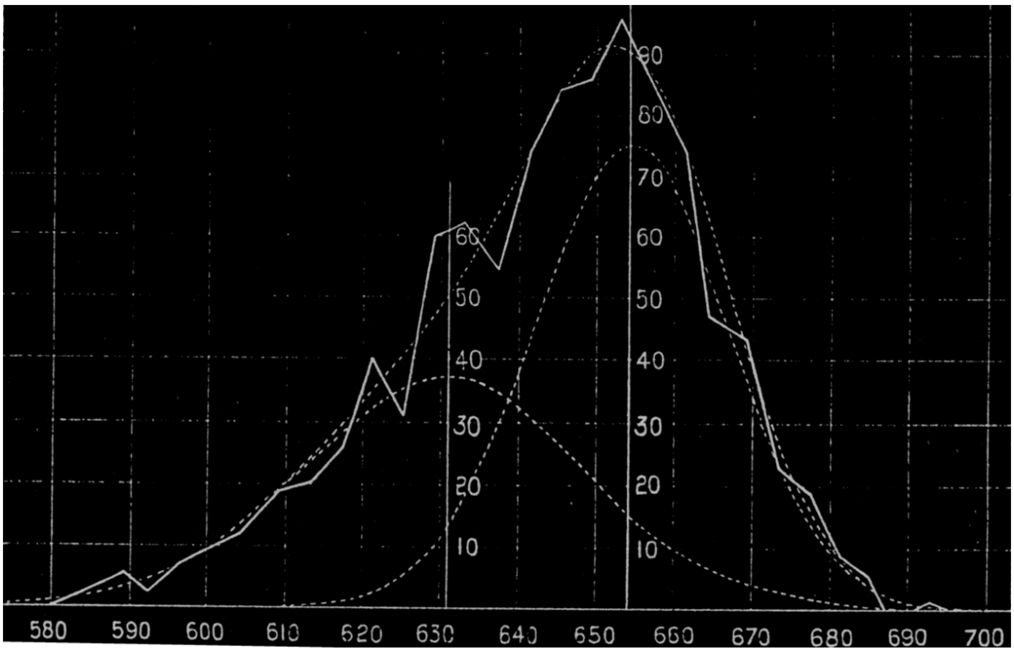

Figure 8.16: Weldon's Diagram of the Crab Dimorphism (1893)

Weldon's crab studies proved to be the cradle from which mathematical statistics would emerge, largely in the hands of Karl Pearson. In fact, an abstract of Pearson's seminal paper 'Contributions to the Mathematical Theory of Evolution' followed Weldon's paper immediately in the *Proceedings of the Royal Society*.[121] A new era was beginning in the history of statistics.

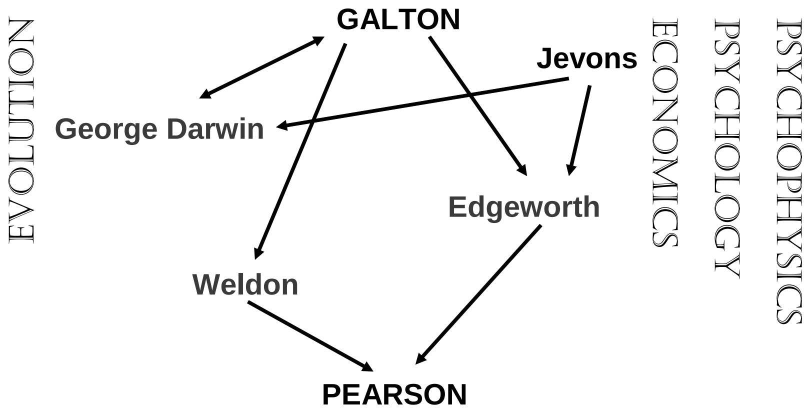

Figure 8.17 Schematic showing associations and influences

Galton's influence on Pearsonian statistics is a classic example of partial causality, indirect and moderate, but it acted directly and reciprocally on George Darwin and was felt strongly, if incompletely, by both Pearson's intellectual predecessors, Edgeworth and Weldon.

Notes

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1 George Darwin, Letter to Francis Galton (4 January 1875, but wrongly dated 1874), UCL Galton Archive, 190A.

2 Francis Galton, Letter to Samuel Tertius Galton (6 December 1839), in Karl Pearson, *Life, Letters and Labours of Francis Galton* Vol I, (hereinafter referred to as *Life of Galton*) 110.

3 A comprehensive and authoritative account of William Hopkins' career as a private tutor, the Mathematical Tripos for which he prepared students and the reform of mathematics at Cambridge University in the nineteenth century are given in Alex D. D. Craik, *Mr Hopkins' Men: Cambridge Reform and British Mathematics in the 19th Century*, Springer-Verlag, London, 2008.

4 Francis Galton, Letter to Samuel Tertius Galton (11 November 1841), in Pearson, *Life of Galton*, I, 163.

5 'Poll men' was the term used for the *hoi polloi*, those who failed to reach the standard required for an honours degree in mathematics (over 60% in the period around Galton's time at Cambridge). See Craik's *Mr Hopkins' Men*, 14, 90, 108.

6 Francis Galton, *Memories of My Life* (London: Methuen, 1908), 304. Further details may be found in Christopher B Pritchard, 'The Normal Curve of Evolutionary Biology, 1869-1877, with Special Reference to the Support Given to Francis Galton by George Darwin', PhD Thesis, Open University (2005), 96-102. (Hereinafter, this work is referenced as Pritchard, Thesis.)

7 Francis Galton, *Hereditary Genius,* 2nd edition (London: Macmillan,1892), 12-32, 325-348.

8 Francis Galton, 'Hereditary Improvement', *Fraser's Magazine* 7 (1873), 116-130, 120.

9 Francis Galton, 'Barometric Predictions of Weather', *Report of the British Association for the Advancement of Science, 1870* (London, 1871), 31-33; reprinted in *Nature* 2 (20 October 1870), 501-503.

10 Francis Darwin, 'Memoir of Sir George Darwin', in *Scientific Papers by Sir George Darwin*, 5 vols. (Cambridge University Press, 1907-16), vol. 5 (1916), ix-xxxiii.

11 Pritchard, Thesis, 338-343.

12 Galton was the inaugural President of the Eugenics Society from 1907. Leonard Darwin (1850-1943) succeeded him in 1911 and continued in this position to 1928.

[13] George Darwin, 'On Beneficial Restriction to Liberty of Marriage', *Contemporary Review* 22 (1873), 412-426.

[14] Francis Galton, 'Hereditary Improvement', *Fraser's Magazine* 7 (1873), 116-130, 123.

[15] George Darwin, 'On Beneficial Restriction to Liberty of Marriage', *Contemporary Review* 22 (1873), 412-426, 414.

[16] Henry William Watson and Francis Galton, 'On the Probability of the Extinction of Families', *Journal of the Anthropological Institute of Great Britain and Ireland* 4 (1874), 138-144. Watson's discussion of the problem also appeared in the *Educational Times* 26 (1873), 115.

[17] Francis Galton, Letter to George Darwin, 3 October 1873, UCL Galton Archive, 245/6; Pearson, *Life of Galton* IIIB, 461-462.

[18] Charles Darwin, 'On the Males and Complemental Males of Certain Cirripedes, and on Rudimentary Structure', *Nature* 8 (25 September 1873), 431-432.

[19] George Darwin, 'Variations of Organs', *Nature* 8 (16 October 1873), 505.

[20] 'Report of a Meeting of the (London) Mathematical Society' (9 April 1874), *Nature* 9 (23 April 1874), 494.

[21] George Darwin, 'Marriages Between First Cousins in England and their Effects', with discussion, *Journal of the Statistical Society* 38 (1875), 153-184, and *Fortnightly Review*, n.s. 18 (1875), 22-41. Idem, 'Note on the Marriages of First Cousins', *Journal of the Statistical Society* 38 (1875), 344-348. The note added little to the thesis of the paper.

[22] See Charles Darwin, Letter to George Darwin, 6 December 1874, Cambridge University Library, DAR 210.1.2.

[23] Francis Galton, 'On a Proposed Statistical Scale', *Nature* 9 (5 March 1874), 342-343, 342.

[24] Ibid., 343. Galton carelessly omitted +3° and -3°, listing five 'graduations' but seven 'places'.

[25] Francis Galton, 'Statistics by Intercomparison', *Philosophical Magazine* [4] 49 (1875), 33-46. For a detailed account of the emergence of the ogive, see Chapter 11 in this volume.

[26] For George Darwin's criticism of Galton's original term, see Francis Galton, Letter to George Darwin, 8 January 1875, UCL Galton Archive, 190A.

[27] Pritchard, Thesis, 264-270. Alexander Bain (1818-1903) was an early proponent of scientific psychology and with John Mill was a major protagonist of the British school of empiricism. Galton made a number of contributions to *Mind*, the first English-medium psychology journal, founded by Bain in 1876.

[28] William Stanley Jevons, *The Theory of Political Economy* (London: Macmillan, 1871).

[29] The hedonistic calculus was a feature of the utilitarianism of Jeremy Bentham (1748-1832), who not only argued that pleasure and pain were quantifiable but identified factors which might be taken into account in any such attachment of a value.

[30] George Darwin, 'Commodities versus Labour', *Contemporary Review* 22 (1873), 689-698.

[31] William Stanley Jevons, Letter to George Darwin, 24 November 1874; reproduced in William Stanley Jevons, *Papers and Correspondence of William Stanley Jevons*, edited by Robert Dennis Collison Black, 7 vols. (London: Macmillan, 1972-81), vol. 4, 81.

[32] George Darwin, 'The Theory of Exchange Value', *Fortnightly Review* 17 (February 1875), 243-253; reprinted in *William Stanley Jevons: Critical Assessments*, edited by John Cunningham Wood, 3 vols. (London: Routledge, 1988), vol. 2, 89-99.

[33] Alexander Bain, *The Emotions and the Will* (London: John W. Parker, 1859).

[34] William Stanley Jevons, *The Theory of Political Economy* (London: Macmillan, 1871), vii-viii.

[35] Ibid., 9.

[36] Ibid., 9-10.

[37] Ibid., 13-14.

[38] Ibid., 20.

[39] Ibid., 96.

[40] Francis Galton, Letter to George Darwin, 9 December 1874, UCL Galton Archive, 245/6.

[41] Francis Galton, Letter to George Darwin, 25 December 1874, Galton Archive, 245/6; reproduced in Pearson, *Life of Galton*, IIIB, 463.

[42] George Darwin, Letter to Francis Galton, 4 January 1875, UCL Galton Archive, 190A.

[43] Francis Galton, Letter to George Darwin, 30 January 1876, UCL Galton Archive, 245/6.

[44] Francis Galton, 'Regression Towards Mediocrity in Hereditary Stature', *Journal of the Anthropological Institute of Great Britain and Ireland* 15 (1886), 246-263 (Table VI, appendix, 260); Pearson recalculated the mean diameters of each group of filial seeds, together with their standard deviations. See Pearson, *Life of Galton*, IIIA, footnote, 7.

[45] Francis Galton, Letter to George Darwin, 10 May 1876, UCL Galton Archive 245/6.

[46] Francis Galton, Letter to George Darwin, 17 May 1876, UCL Galton Archive 245/6.

[47] Ibid.

[48] Francis Galton, Letter to George Darwin, 5 January 1877, UCL Galton Archive 245/6.

[49] Ibid.

[50] Charles Darwin, Letter to Francis Galton, 9 January 1877, UCL Galton Archive 39.

[51] Francis Galton, Letter to George Darwin, 12 January 1877, UCL Galton Archive 245/6; Pearson, *Life of Galton*, IIIB, 465-466.

[52] Francis Galton, Letter to Charles Darwin, 12 January 1877, in Pearson, *Life of Galton*, II, 192.

[53] Francis Galton, 'Typical Laws of Heredity', *Nature* 15 (1877), 492-495, 512-514, 532-533.

[54] Francis Galton, Letter to George Darwin, 12 January 1877, UCL Galton Archive 245/6; Pearson, *Life of Galton*, IIIB, 465-466.

[55] George Darwin's years of transition, leading to his development into one of the leading mathematical astronomers of his day, are detailed in Pritchard, Thesis, 325-350.

[56] Francis Galton, 'Statistics of Mental Imagery', *Mind* 5 (1880), 301-318.

[57] Ibid., 301.

[58] Francis Galton, 'Mental Imagery', *Fortnightly Review* n.s. 28 (1880), 312-324.

[59] The evening discourse was delivered on 30 August 1880. See *Report of the 50th Meeting of the British Association for the Advancement of Science, 1880* (London, 1882), lxxviii.

[60] Galton, 'Statistics of Mental Imagery', 306.

[61] Francis Galton, 'Report of the Anthropometric Committee', *Report of the 51st Meeting of the British Association for the Advancement of Science, 1881* (London, 1882), 244-260.

[62] Francis Ysidro Edgeworth, Letter to Francis Galton, (16 February 1888 / 1889?), UCL Galton Archive, 237.

[63] Stephen M. Stigler, A History of Statistics: The Measurement of Uncertainty Before 1900, Cambridge, Mass.: Harvard University Press (Belknap), 1986, 307. In Stigler's History, the chapter on Edgeworth builds on his earlier paper, 'Francis Ysidro Edgeworth, Statistician', *JRSS*, A, 141 (1978), 287-322.

[64] Philip Mirowski (ed.), *Edgeworth on Chance, Economic Hazard, and Statistics*, Lanham, MD: Rowman & Littlefield, 1994, 9.

[65] John Creedy, 'F. Y. Edgeworth's mathematical training', *Journal of the Royal Statistical Society* [A] 146 (1983), 158-162.

[66] John Creedy, *Edgeworth and the Development of Neoclassical Economics*, Oxford: Blackwell, 1986.

[67] The papers, held in the Pearson Archive at UCL have been studied by Flatau. Paul Flatau, 'Jevons's one great disciple: Wicksteed and the Jevonian Revolution in the second generation', at http://hetsa.fec.anu.edu.au/review/ejournal/pdf/40-A-4.pdf

[68] Francis Ysidro Edgeworth, *New and Old Methods of Ethics, or "Physical Ethics" and "Methods of Ethics"*, Oxford & London: James Parker (1877), 55.

[69] Ibid., 464.

[70] Ibid., iii.

[71] Francis Galton, *Hereditary Genius*, London: Macmillan (1869), 362. Edgeworth, *Method of Ethics*, 72.

[72] Pritchard, Thesis, Chapter 4.

[73] Theodore M. Porter, *The Rise of Statistical Thinking, 1820-1900*, Princeton University Press, 1986, 256-257. Philip Mirowski (ed.), *Edgeworth on Chance, Economic Hazard, and Statistics*, Lanham, MD: Rowman & Littlefield, 1994, 1-79.

[74] Francis Ysidro Edgeworth, *New and Old Methods of Ethics*, 8-9.

[75] Ibid., 10.

[76] Ibid., 12-13.

[77] Philip Mirowski (ed.), *Edgeworth on Chance, Economic Hazard, and Statistics*, Lanham, MD: Rowman & Littlefield, 1994, 28.

[78] William Stanley Jevons, 'Review of "Mathematical Psychics" ', *Mind* 6 (1881), 581-583; 583.

[79] Theodore M. Porter, *The Rise of Statistical Thinking, 1820-1900*, Princeton University Press, 1986, 255. Philip Mirowski (ed.), *Edgeworth on Chance, Economic Hazard, and Statistics*, Lanham, MD: Rowman & Littlefield, 1994, 5.

[80] Stephen M. Stigler, *Statistics on the Table: The History of Statistical Concepts and Methods*, Cambridge, Mass.: Harvard University Press, 1999, 95.

[81] Francis Galton, Letter to Francis Ysidro Edgeworth (28 October 1881), in Stigler (1986), 306-307. Later and for a different audience he was more critical, writing 'Edgeworth … fails in sustained clearness of expression. He is moreover somewhat over fond of using higher and more mathematics than is always necessary.' See Francis Galton, Letter to William Fleetwood Sheppard, 24 October 1892, in Pearson, *Life of Galton*, IIIB, 486. The bulk of the correspondence between Galton and Edgeworth is held in Folder 237 of the Galton Archive at University College London, and consists of 25 undated letters, catalogued as 1887?-1896. Additional material is held in Folders 133/8E, 138/7, 152/4, 189.

[82] Edgeworth, *New and Old Methods of Ethics*, 44.

[83] Francis Ysidro Edgeworth, 'The Element of Chance in Competitive Examinations', *JSS* 53 (1890), 460-475, 472.

[84] Ibid., 473.

[85] Stephen M. Stigler, 'Francis Ysidro Edgeworth, Statistician', *JRSS* [A] 141 (1978), 287-322, especially 292-293.

[86] Francis Ysidro Edgeworth, 'On Methods of Ascertaining Variations in the Rate of Births, Deaths and Marriages', *JRSS* 48 (1885), 628-649, 639.

[87] Edgeworth, *New and Old Methods of Ethics*, 19.

[88] Ibid., 40.

[89] Philip Mirowski (ed.), *Edgeworth on Chance, Economic Hazard, and Statistics*, Lanham, MD: Rowman & Littlefield, 1994, 19.

[90] Francis Ysidro Edgeworth, 'The Method of Least Squares', *Philosophical Magazine*16 (1883), 360-375; Idem, 'The Philosophy of Chance', *Mind* 9 (1884), 223-235; Idem, 'A priori Probabilities', *Philosophical Magazine* 18 (1884), 204-210; Idem, 'Chance and Law', *Hermathena* 5 (1884), 154-163.

[91] Francis Ysidro Edgeworth, 'Observations and Statistics: An Essay on the Theory of Errors of Observation and the First Principles of Statistics', *Transactions of the Cambridge Philosophical Society* 14 (1887), 138-169; Idem, 'Methods of Statistics', *Jubilee Volume of the Statistical Society*, London: Edward Stanford, 1885, 181-217; Idem, 'On Methods of Ascertaining Variation in the Rate of Births, Deaths, and Marriages', *JSS* 48 (1885), 628-652; Idem, 'Progressive Means', *JSS* 49 (1886), 469-475. The four papers are listed here in the order in which the papers were read, rather than published, respectively, May, June, September and October 1885.

[92] Stephen M. Stigler, *A History of Statistics: The Measurement of Uncertainty Before 1900*, Cambridge, Mass.: Harvard University Press (Belknap), 1986, 308-315; Idem, *Statistics on the Table: The History of Statistical Concepts and Methods*, Cambridge, Mass.: Harvard University Press, 1999, 413-414, especially 100-104.

[93] Francis Galton, 'Regression towards Mediocrity in Human Stature', *Journal of the Anthropological Institute* 15 (1886), 246-263.

[94] Francis Ysidro Edgeworth, 'The Statistics of Examinations', *JSS* 51 (1888), 599-635, 621.

[95] Ibid., 625.

[96] Francis Ysidro Edgeworth, 'The Mathematical Theory of Banking', *JSS* 51 (1888), 113-127, 117.

[97] Francis Ysidro Edgeworth, 'Miscellaneous Applications of the Calculus of Probabilities, (Continued)', *JSS* 61 (1898), 119-131 & 534-544.

[98] Ibid., 534.

[99] An elaborated version was given at the Royal Society on 21 January 1886. Francis Galton, 'Presidential Address, Section H', *British Association Report* 55 (1885), 1206-1214; Ibid., 'Family likeness in stature', *Proceedings of the Royal Society* 40 (1886), 42-73.

[100] Francis Galton, 'Regression towards Mediocrity in Hereditary Stature', *Journal of the Anthropological Institute* 15 (1886), 246-263; Ibid., 'Family Likeness in Eye-colour, *Proceedings of*

the Royal Society 40 (1886), 402-416; Ibid., 'Co-relations and their Measurement, Chiefly from Anthropological Data', *Proceedings of the Royal Society* 45 (1888), 135-145; Ibid., 'President's Address', *Journal of the Anthropological Society*, 18 (1889), 401-419; Ibid., *Natural Inheritance*, London: Macmillan (1889); Ibid., 'Kinship and Correlation', *North American Review* 150 (1890), 419-431. Francis Ysidro Edgeworth, Newmarch Lecture, University College London (1 June 1892); Ibid., 'Correlated Averages', *Philosophical Magazine* [5] 34 (1892), 190-204; Ibid., 'The Law of Error and Correlated Averages', *Philosophical Magazine* [5] 34 (1892), 429-438, 518-526; Ibid., 'A New Method of Treating Correlated Averages', *Philosophical Magazine* [5] 35 (1893), 63-64; Ibid., 'Exercises in the Calculation of Errors', *Philosophical Magazine* [5] 36 (1893), 98-111; Ibid., 'Note of the Calculation of Correlation between Organs', *Philosophical Magazine* [5] 36 (1893), 350-351; Ibid., 'Statistical Correlation between Social Phenomena', *Journal of the Royal Statistical Society* 56 (1893), 670-675; Ibid., 'Asymmetrical Correlation between Social Phenomena', *Journal of the Royal Statistical Society* 57 (1894), 563-568.

[101] George Howard Darwin, 'On Fallible Measures of Variable Quantities, and on the Treatment of Meteorological Observations', *Philosophical Magazine* [5] 4 (1877), 1-14. Francis Galton, 'The Application of a Graphic Method to Fallible Measures', *Jubilee Volume of the [Royal] Statistical Society* (1885), 262-265; discussion, 266-271.

[102] Stigler, *History of Statistics* (1986), 299.

[103] Francis Ysidro Edgeworth, Letter to Francis Galton (15 May 1892), UCL Galton Archive 237.

[104] Stigler, *Statistics on the Table* (1999), 107. In October, Edgeworth wrote 'once more let me thank you for your excellent advice' and in December he referred to 'our conversation last night'. See Francis Ysidro Edgeworth, Letters to Francis Galton (29 October 18982, 3 December 1892), UCL Galton Archive, 237.

[105] Following the fifth lecture Edgeworth wrote that 'the quincunx worked splendidly and imparted distinction to my discourse. I will use the apparatus once more next Wednesday, if not receiving a veto.' We know, then, only of Edgeworth's intention of using the quincunx on the second occasion. Francis Ysidro Edgeworth, Letter to Galton (9 June 1892), UCL Galton Archive, 237.

[106] Weldon is known to have been taught by Stephen Parkinson (1823-1889), who beat William Thomson to be Senior Wrangler in 1845.

[107] Francis Darwin, 'Memoir of Sir George Darwin', *Scientific Papers by Sir George Darwin*, 5 vols. (Cambridge University Press, 1907-1916), vol. 5, ix-xxxiii, xiii.

[108] Karl Pearson, 'Walter Frank Raphael Weldon, 1860-1906. A Memoir', *Biometrika* 5(1906), 1-52, 2.

[109] W. F. R. Weldon, 'The Variations occurring in certain Decapod Crustacea, I. *Crangon vulgaris*', *Proceedings of the Royal Society* 47 (1890), 445-453, 445.

[110] The committee was formed at a meeting at the Savile Club on 9 December 1893, and met for the first time under Galton's chairmanship on 25 January 1894. Its first report was W. F. R. Weldon, 'Report of the Committee … for Conducting Statistical Inquiries into the Measurable Characteristics of Plants and Animals. Part 1: An Attempt to Measure the Death-rate due to the Selective Destruction of *Carcinus Mœnas*', *Proceedings of the Royal Society* 57 (1895), 360-382.

[111] W. F. R. Weldon, 'The Variations occurring in certain Decapod Crustacea, I. *Crangon vulgaris*', *Proceedings of the Royal Society* 47 (1890), 445-453.

[112] Ibid., 445.

[113] The Weldon-Galton correspondence is held in folders 340A-J in the Galton Archive, University College London. The first folder contains 179 items, constituting most but not all of the letters sent in 1890 and 1891.

[114] Weldon, 'The Variations occurring in certain Decapod Crustacea, I. *Crangon vulgaris*', 446.

[115] Ibid.

[116] W. F. R. Weldon, 'Certain Correlated Variations in *Crangon vulgaris*', *Proceedings of the Royal Society* 51 (1892), 2-21.

[117] Francis Galton, 'Co-relations and their Measurement, chiefly from Anthropometric Data', *Proceedings of the Royal Society* 45 (1888), 135-145.

[118] Karl Pearson, 'Walter Frank Raphael Weldon, 1860-1906. A Memoir', *Biometrika* 5(1906), 1-52; reprinted in E. S. Pearson & M. G. Kendall (eds.), (1970), *Studies in the History of Statistics and Probability*, London: Griffen, 1970, 264-321, 283.

[119] W. R. F. Weldon, 'On certain Correlated Variations in *Carcinus mœnas*', *Proceedings of the Royal Society* 54 (1893), 318-329.

[120] Ibid., 325. For a discussion of attempts to model the distribution of variation in Naples crabs, see M. Eileen Magnello, 'Karl Pearson's Gresham Lectures: W.F.R. Weldon, speciation and the origins of Pearsonian statistics'. *British Journal for the History of Science,* 29 (1996), 43-63.

[121] Karl Pearson, Abstract of 'Contributions to the Mathematical Theory of Evolution', *Proceedings of the Royal Society* 54 (1893), 329-333.