2. *On the Controversy concerning the Seat of Volta's Contact Force.*[1] *By* Professor OLIVER LODGE, *F.R.S.*

MONDAY, SEPTEMBER 18.

The Section was divided into two Departments.

The following Reports and Papers were read:

DEPARTMENT I.—MATHEMATICS.

1. *Report on Tables of certain Integrals.* See Reports, p. 65.

2. *Report on Tables of certain Mathematical Functions.* See Reports, p. 160.

3. *The Median Estimate.* *By* FRANCIS GALTON, *D.C.L., F.R.S.*

The usual method is very unsatisfactory by which the collective opinion of Councils, Senates, and other Assemblies is ascertained, in respect to the most suitable amount of money to be granted for any particular purpose. The opinions of individual members are sure to differ as to rewards for past services, as to compensation for damage, or as to the cost of carrying out some desirable object for which provision has to be made. How is that medium amount to be ascertained which is the fairest compromise between many different opinions? The method usually adopted is for some person in authority to consult his colleagues and then to lay a definite proposal before the meeting, to which another person may move an amendment; the amendment and the original motion are then put severally to the vote, and are carried or rejected by a simple majority. Jurymen are said to adopt a different way of assessing damages; each writes his own estimate on a separate paper, the estimates are added together, and the average of them all is occasionally accepted by the whole body of the jury and returned as their verdict. Averages are, however, objectionable to large assemblages on account of the tedious arithmetic that would then be needed. Moreover, an average value may greatly mislead, unless each several estimate has been made in good faith, because a single voter is able to produce an effect far beyond his due share by writing down an unreasonably large or unreasonably small sum. The middlemost value, or the *median* of all the estimates, is free from this danger, inasmuch as the influence of each voter has exactly equal weight in its determination. Again, few persons know what they want with sufficient clearness to enable them to express it in numerical terms, from which alone an average may be derived. Much deeper searching of the thought is needed to enable a man to make such precise affirmation as that 'in my opinion the bonus to be given should be 80*l*.,' than to enable him to say, 'I do not think the bonus should be so much as 100*l*., certainly it should not be more than 100*l*.'

The plan that I would suggest for discovering the *median* of the various sums desired by the several voters is to specify any two reasonable amounts, A and B, A being the smaller, making it understood that A and B are intended to serve as *divisions*, and therefore no votes are to be given for either of those two precise

[1] This paper will be published in the *Proceedings of the Physical Society of London.*

sums. Next, three shows and counts of hands are to be made: (1) for less than A; (2) for more than A, but less than B; (3) for more than B. The results are

a per cent. vote for less than A; 100–a vote for more than A.
b per cent. vote for less than B; 100–b vote for more than B.

Numerous analogies amply justify the assumption that the estimates will be distributed on either side of their (unknown) median, m, with an (unknown) quartile, q, in approximate accordance with the normal law of frequency of error. The following table of centiles (a better word than 'Per-centiles,' which I originally used), having a quartile = 1, is founded upon that law. It is extracted from my 'Natural Inheritance' (Macmillan, 1889, p. 205) to serve the present purpose.

Centiles to the Grades 0°–100°.

Grades	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
0°	−inf:	−3·45	−3·05	−2·79	−2·60	−2·44	−2·31	−2·19	−2·08	−1·99
10°	−1·90	−1·82	−1·74	−1·67	−1·60	−1·54	−1·47	−1·42	−1·36	−1·30
20°	−1·25	−1·20	−1·15	−1·10	−1·05	−1·00	−0·95	−0·91	−0·86	−0·82
30°	−0·78	−0·74	−0·69	−0·65	−0·61	−0·57	−0·53	−0·49	−0·45	−0·41
40°	−0·38	−0·34	−0·30	−0·26	−0·22	−0·19	−0·15	−0·11	−0·07	−0·04
50°	0·00	+0·04	+0·07	+0·11	+0·15	+0·19	+0·22	+0·26	+0·30	+0·34
60°	+0·38	+0·41	+0·45	+0·49	+0·53	+0·57	+0·61	+0·65	+0·69	+0·74
70°	+0·78	+0·82	+0·86	+0·91	+0·95	+1·00	+1·05	+1·10	+1·15	+1·20
80°	+1·25	+1·30	+1·36	+1·42	+1·47	+1·54	+1·60	+1·67	+1·74	+1·82
90°	+1·90	+1·99	+2·08	+2·19	+2·31	+2·44	+2·60	+2·79	+3·05	+3·45

Let α be the tabular number *inclusive of its sign*, that corresponds to the grade $a°$, and let β be that which corresponds to $b°$, then

$$m + q\alpha = \text{A}\,;\ m + q\beta = \text{B},$$

whence

$$m = \text{A} - \alpha\left\{\frac{\text{B}-\text{A}}{\beta-\alpha}\right\} = \text{B} - \beta\left\{\frac{\text{B}-\text{A}}{\beta-\alpha}\right\}$$

Example:—A = 100, B = 500; $a = 40°$, $b = 80°$, whence $\alpha = -0·38$, $\beta = +1·25$ and $m = 193$. The truth of the determination of m may now, if so desired, be tested by putting two new values A′ B′ to the vote, in the same way as A and B, but A′ and B′ should not differ much from m, and it should be an honourable understanding that no member should deviate from his first opinion in giving his second vote.

When about to utilise this method, A and B ought to be so selected that A shall secure not less than 5 per cent. of the votes, and B not more than 95, because the curve of error ceases to be trustworthy near to its extremities, but a dependence upon it within the limits of 5° and 95° will seem pedantic only to those who are unfamiliar with its nature and with its numerous and successful applications.

It will be easily understood that this method is a particular case of the more general problem, that in any system of normal variables which has been arrayed between the grades of 0° and 100°, if the values be given that correspond to *any two* specified grades, those that correspond to each and every other grade can be found.

I heartily wish that when occasion offers, some Assembly may be disposed to experiment on the above method. The calculators should, of course, rehearse the work beforehand, and be well prepared to carry it through both rapidly and surely.

It is worth mentioning that when the above table is not at hand, a graphical substitute for it, that ranges between 5° and 95° and is true to the first place of decimals, may be quickly made by those who can recollect three simple factors. Thus, draw between two vertical limits, 0° and 100°, a straight line on squarely ruled paper, having a quartile equal to 1. Accept this line in lieu of the curve between 30° and 70°, add one-twentieth to the lengths of the centiles at 20° and 80°, one-fifth to those at 10° and 90°, and one-third to those at 50° and 95°. Then unite the tops of these centiles with a free-hand curve.

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4. *A System of Invariants for Parallel Configurations in Space.* *By* Professor A. R. Forsyth, *Sc.D.*, *F.R.S.*

There is one class of invariants appertaining to parallel configurations which I have not seen noticed; they arise in spaces of two, three, and any number of dimensions.

It is known that, for a plane curve parallel to a given plane curve, the normals at corresponding points are the same in direction, and therefore the angle between corresponding consecutive normals is the same, so that this infinitesimal angle is an invariantive element. Moreover, the centres of curvature are the same, so that the difference between the radii of curvature is the diameter of the rolling circle, the two enveloping curves of which are the given curve and its parallel.

Similarly, in the case of parallel surfaces, it is convenient to consider the principal directions of curvature at each point. They are respectively parallel to one another at corresponding points; the corresponding normals are coincident in direction: and the centres of principal curvatures for the two surfaces are the same. Hence the difference between a principal radius of curvature and the corresponding principal radius of curvature of the parallel surface is equal to the diameter of the rolling sphere, the other envelope of which is the parallel surface; and this holds for each of the two principal radii.

Likewise, in space of n dimensions. To render the explanations clearer, we consider a configuration

$$F(x_1, x_2, \ldots x_n) = 0,$$

which, as for two and for three dimensions, will be considered devoid of special singularities. Let

$$l_s = \frac{\delta F}{\delta x_s} \div \Delta, \quad (s = 1, \ldots, n),$$

where

$$\Delta^2 = \sum_{s=1}^{n} \left(\frac{\delta F}{\delta x_s}\right)^2.$$

If, then, a distance ρ be measured inwards along the direction indicated by $l_1, \ldots, l_n$ (say along the normal to the surface $F = 0$), the coordinates of the extremities are given by

$$\xi_s = x_s - \rho l_s.$$

If this point be a point of intersection with a consecutive normal at $x_1 + dx_1, \ldots, x_n + dx_n$, then

$$\xi_s = x_s + dx - \rho(l_s + dl),$$

for $s = 1, \ldots n$; that is,

$$0 = \left(\frac{1}{\rho} - \frac{\partial l_s}{\partial x_s}\right) dx_s - \sum_{i=1}^{n} \frac{\partial l}{\partial n_i} dx_i,$$

the term in dx_s being omitted from the summation on the right-hand side, and the equation holding for $s = 1, \ldots n$. The possible values of ρ are given by the equation

$$\begin{vmatrix} \frac{1}{\rho} - \frac{\partial l_1}{\partial x_1}, & \frac{\partial l_1}{\partial x_2}, & \frac{\partial l_1}{\partial x_3}, & \ldots \\ \frac{\partial l_2}{\partial x_1}, & \frac{1}{\rho} - \frac{\partial l_2}{\partial x_2}, & \frac{\partial l_2}{\partial x_3}, & \ldots \\ \frac{\partial l_3}{\partial x_1}, & \frac{\partial l_3}{\partial x_2}, & \frac{1}{\rho} - \frac{\partial l_3}{\partial x_3}, & \ldots \\ \cdot & \cdot & \cdot & \cdot \end{vmatrix} = 0$$