

cation by Dalzell in 1861 of his "Flora of Bombay." It is impossible in a brief review like the present to mention the names of all the workers who, in various parts of the gradually extending Indian Empire, added to our knowledge of its botanical wealth. It must suffice to mention the names of a few of the chief, such as Hardwicke, Madden, Munro, Edgeworth, Lance and Vicary, who collected and observed in Northern India, and who all, except the two last mentioned, also published botanical papers and pamphlets of more or less importance; Jenkins, Masters, Mack, Simons and Oldham, who all collected extensively in Assam; Hofmeister, who accompanied Prince Waldemar of Prussia, and whose collections form the basis of the fine work by Klotsch and Garcke (*Reis. Pr. Wald.*); Norris, Prince, Lobb and Cuming, whose labours were in Penang and Malacca; and last, but not least, Strachey and Winterbottom, whose large and valuable collections, amounting to about 2000 species, were made during 1848 to 1850 in the higher ranges of the Kamaon and Gharwal Himalaya, and in the adjacent parts of Tibet. In referring to the latter classic Herbarium, Sir Joseph Hooker remarks that it is "the most valuable for its size that has ever been distributed from India." General Strachey is the only one who survives of the splendid band of collectors whom I have mentioned. I cannot conclude this brief account of the botanical labours of our first period without mentioning one more book, and that is the "Hortus Calcuttensis" of Voigt. Under the form of a list, this excellent work, published in 1845, contains a great deal of information about the plants growing near Calcutta, either wild or in fields and gardens. It is strong in vernacular names and vegetable economics.

(To be continued.)

#### MATHEMATICS AT THE BRITISH ASSOCIATION.

THE visit of the French Association to Dover necessitated some departures from the usual programme of the British Association week, and the mathematical meeting was held this year on Monday, September 18. Prof. Forsyth, of Cambridge, presided over a well-filled room.

The session opened with the formal communication of two reports of committees: the first, drawn up by Prof. Karl Pearson, and practically forming a continuation of a previous report, contains a set of tables of certain functions connected with the integral

$$G(r, \nu) = \int_0^{\pi} \sin^{\nu} \theta e^{r \cos \theta} d\theta,$$

for integral values of  $r$  from 1 to 50, and for values of  $\nu$  at certain intervals from 0 to 1. These functions are of importance in certain statistical problems.

The second report consists substantially of the new "Canon Arithmeticus" which Lieut.-Colonel Cunningham has prepared; the Association has made a grant for publishing the tables as a separate volume (they cannot well be fitted into the comparatively small page of the B.A. Report), and it is to be hoped that before long they will become generally available for workers in the Theory of Numbers.

The first of the papers was read by Dr. Francis Galton, on "The Median Estimate." Dr. Galton proposes to substitute a scientific method for the very unsatisfactory ways in which the collective opinion of committees and assemblies of various kinds is ascertained, in respect to the most suitable amount of money to be granted for any particular purpose. How is that medium amount to be ascertained which is the fairest compromise between many different opinions? An average value—*i.e.* the arithmetic mean of the different estimates—may greatly mislead, because a single voter is able to produce an effect far beyond his due share by writing down an unreasonably large or unreasonably small sum. Again, few persons know what they want with sufficient clearness to enable them to express it in numerical terms, from which alone an average may be derived; much deeper thought-searching is needed to enable a man to make such a precise affirmation as that "in my opinion the bonus to be given should be 80%," than to enable him to say "I do not think he deserves so much as 100%, certainly not more than 100%."

Dr. Galton's plan for discovering the medium of the various sums desired by the several voters is to specify any two reasonable amounts A and B, and to find what percentage  $a$  of voters think that the sum ought to be less than A, and what percentage  $b$  vote for less than B. It may now be assumed that

the estimates will be distributed on either side of their (unknown) median  $m$ , with an (unknown) quartile  $q$ , in approximate accordance with the normal law of frequency of error; and then (using the table of centiles given in the author's "Natural Inheritance") the required median value can be found.

This was followed by a paper "On a system of invariants of parallel configurations in space," by Prof. Forsyth. The process followed by the author is one in which English mathematicians have always excelled—namely, the deduction of difficult analytical results from simple geometrical considerations. Prof. Forsyth's final formulæ may be regarded as invariants of relations between certain definite integrals; the way in which he finds them is as follows:—

Consider any plane curve; if we suppose a circle of constant size to roll on the curve, its envelope will be another curve, which is said to be *parallel* to the original one. If now  $L$  be the length and  $A$  the area of a curve, it is found that the quantity  $A - \frac{1}{4\pi}L^2$  has the same value for the parallel as for the original curve; in other words,

$$A - \frac{1}{4\pi}L^2$$

is *invariantive* for parallel curves. Similarly in space of three dimensions, the envelope of a sphere of fixed size which rolls on a given surface is another *parallel* surface; and if  $V$  be the volume contained by a surface,  $S$  its superficial area, and  $L$  twice the surface-aggregate of the mean of the curvatures at any point, then it is found that the quantities

$$S - \frac{1}{16\pi}L^2 \text{ and } V - \frac{1}{8\pi}LS + \frac{1}{192\pi^2}L^3$$

are invariantive for all parallel surfaces.

Similar results hold for space of  $n$  dimensions. At the end of the paper the expressions obtained are shown to be connected with the ordinary invariant-theory of binary forms.

The next paper, read by Prof. Everett, was concerned with "The Notation of the Calculus of Differences." In conjunction with the ordinary symbol  $\Delta$ , defined by

$$\Delta y_n = y_{n+1} - y_n,$$

Prof. Everett employs another symbol  $\delta$ , defined by

$$\delta y_n = y_n - y_{n-1},$$

so that

$$\delta = \Delta / (1 + \Delta).$$

The use of  $\delta$  simplifies some of the well-known formulæ of the calculus of finite differences.

Prof. A. C. Dixon, of Galway, followed, with a paper "On the Partial Differential Equation of the Second Order." Let  $z$  be the dependent, and  $x$  and  $y$  the independent, variables, and with the usual notation, let

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}, \quad r = \frac{\partial^2 z}{\partial x^2}, \quad s = \frac{\partial^2 z}{\partial x \partial y}, \quad t = \frac{\partial^2 z}{\partial y^2},$$

and consider the differential equation

$$f(x, y, z, p, q, r, s, t) = 0.$$

This may be supposed solved by using two more relations

$$u = a, \quad v = b,$$

among the quantities  $x, y, z, p, q, r, s, t$ , to give values of  $r, s, t$ , which, when substituted in

$$dz = p dx + q dy, \quad dp = r dx + s dy, \quad dq = s dx + t dy,$$

render these three equations integrable. This will not be possible, of course, unless the expressions  $u, v$ , fulfil certain conditions. Prof. Dixon considers the case in which  $u$  can be so determined that  $v$  is only subjected to one condition, and finds that then  $du$  is a linear combination of the differential expressions used in Hamburger's method of solution. If such a function  $u$  can be found, the system  $f=0, u=a$ , will have a series of solutions depending on an arbitrary function of one variable, and involving two further arbitrary constants.

The next paper, "On the Fundamental Differential Equations of Geometry," was read by Dr. Irving Stringham, of the University of California. Dr. Stringham derives the analytical formulæ for non-Euclidian Geometry by following a procedure indicated by Feyer St. Marie, and later discussed in Killing's "Nicht-Euclidianischen Raumformen." Within an infinitesimal domain in non-Euclidian space the geometrical properties of